Abstract

The aesthetic features of some free form surfaces, such as vases, despite being adjunct, can prove tedious and time consuming to create by the conventional surface models, such as NURBS. In this paper, we propose a new method that is able to rapidly generate complex free form surfaces, such as vases together with their accessory features using the fourth order partial differential equations (PDEs) with three vector-valued shape parameters. Vases of different shapes can be easily produced by altering the shape parameters, force functions and boundary conditions of the proposed PDE. As computational performance is of paramount significance for interactive computer graphics applications, an analytical solution to the PDE has been derived. The effects of the shape parameters, boundary conditions and force functions on the final surface shape have also been studied. © 2002 Published by Elsevier Science Ltd.

Keywords: Vase design; Free form surfaces; Partial differential equations

1. Introduction

Vases are a class of free form surfaces. Primarily they are rotational surfaces. However, the adjunct features prove more important in the representation of aesthetics than their primary shape. Although their degree of beauty can be improved with texture mapping techniques, such as [1], their geometry has more fundamental influence, hence is more important. Due to the complexity and irregularity of the adjunct features, vases are actually quite difficult and tedious to model.

Free form surfaces can be interactively created using many surface modelling methods, such as Bézier and B-spline surfaces. NURBS perhaps is the most popular among such methods, and has been incorporated into a number of modelling packages. However, because of the special characteristics of various vases, a large amount of dedicated manual operations have to be carried out, if these modelling methods are to be used.

The method of surface generation based on the resolution of partial differential equations (PDEs) was first proposed by Bloor and Wilson [2]. In their work, Bloor and Wilson employed fourth order biharmonic PDEs with one vector-valued parameter to create some free form surfaces [3]. In order to solve the PDEs, Brown et al. [4] developed a finite element method (FEM) solution that is used to the generation of complex surfaces. Later, Bloor and Wilson [5] discussed the local control of surfaces generated using PDEs and created the surface of a submarine. More recently, they [6] improved their Fourier series solution by introducing a remainder function into the solution to satisfy the boundary conditions. This method was employed to produce the geometry of an aircraft. Most of Bloor’s work employed numerical solutions to the PDEs, as understandably analytical solutions are generally much harder to obtain than numerical ones. The side effect of most numerical solutions, however, is that they are slow to compute and therefore in many cases are not attractive to interactive surface modelling.

Since the parameters in the PDEs have strong effects on the shape of the surfaces generated with the solutions of the PDEs, You and Zhang [7] have proposed a more general fourth order PDE for blending surface generation. This equation is similar to those used in our previous work [8–10]. It has three vector-valued shape
parameters and covers all forms of existing fourth order PDEs used for free form surface generation. It is, therefore, capable of generating surfaces of much greater variety.

Due to the characteristics of vases, the PDE method appears advantageous over other surface modelers for interactive vase design. However, three issues have to be addressed before this method can be effectively applied. They are: (1) the determination of the form of the PDEs, including the adjustable parameters; (2) the determination of the boundary conditions; and (3) the resolution of the PDEs. Importantly, as analytical expressions of a surface are much more efficient to compute than numerical forms, analytical or semi-analytical solutions are always preferred, despite the fact that the determination of such a solution to PDEs is not usually easy.

The opening and the base profile of a vase are 3D closed curves. For most vases, these closed curves can be described with a periodic function and can be incorporated in the PDE boundary conditions. The objective of this research is to generate the surface of vases or similar objects using the solution of the PDEs.

2. PDEs and their closed form solutions

The solution of a bivariate PDE represents a surface. However, PDEs of different orders have impact both on their computational efficiency and on the capacity of surface generation. In our previous work, we have investigated this problem using the second, fourth and mixed order PDEs [11]. It was found that the higher the order of a PDE, the less efficient and the more powerful it is, and the harder it is to be solved analytically. Taking these factors into account, we believe a fourth order PDE is the most appropriate for our applications.

In vector forms, our proposed fourth order PDEs with three vector-valued shape parameters are mathematically described as

$$
\left( b \frac{\partial ^4}{\partial u^4} + c \frac{\partial ^4}{\partial u^2 \partial v^2} + d \frac{\partial ^4}{\partial v^4} \right) X(u, v) = p(u, v),
$$

where $X = X(u, v)$ represents the generated surface, $b = [b_x, b_y, b_z]^T$, $c = [c_x, c_y, c_z]^T$, and $d = [d_x, d_y, d_z]^T$ are known as the vector-valued shape parameters, $u$ and $v$ are the surface parameters, and $p(u, v)$ is a vector-valued force function.

The boundary conditions can be formulated as the following equations:

$$
\begin{align*}
X(u_0, v) &= g_1(v), \\
X(u_1, v) &= g_2(v), \\
X_x(u_0, v) &= g_3(v), \\
X_x(u_1, v) &= g_4(v).
\end{align*}
$$

The proposed fourth order PDE (1) subject to the boundary conditions (2) can be solved with the power series method [12], or numerical methods such as the FEM [4] and the weighted residual method [13]. On performance ground, we will endeavour to arrive at a closed form solution (analytical solution) making use of our previous work [14].

Since the functional boundary conditions of a vase surface can always be expressed with closed 3D curves, periodic functions are a natural choice. Therefore, the solutions of Eq. (1) under the boundary conditions (2) can be taken as periodic functions of $v$. Notice that the second and fourth derivatives of periodic functions can be formulated by the original functions multiplied by constants, we obtain the following general form of the closed form solution of the homogenous equation of Eq. (1), which will represent the analytical form of the vase surface:

$$
X(u, v) = B_0(u) + \sum_{m=1}^{M} B_m(u)f_m(v),
$$

where $f_m(v)$ consists of the elementary functions of the boundary conditions (2) and

$$
B_0(u) = b_{00} + b_{01}u + b_{02}u^2 + b_{03}u^3,
$$

where $b_{0k}$ ($k = 0, \ldots, 3$) are unknowns, and $M$ stands for the number of elementary functions in the boundary conditions. Depending on the combination of the vector-valued parameters in Eq. (1), $B_m(u)$ has two different forms which are given by

$$
B_m(u) = B_{m1}e^{\alpha_1u} + B_{m2}e^{\alpha_2u} + B_{m3}e^{\alpha_3u} + B_{m4}e^{\alpha_4u}
$$

and

$$
B_m(u) = (B_{m1} + B_{m2}i)e^{\alpha_1u} + (B_{m3} + B_{m4}i)e^{\alpha_2u},
$$

where $B_{mi}$ ($i = 1, \ldots, 4$) are unknowns, and $\alpha_i$ ($i = 1, 2, 3, 4$) are determined by both the vector-valued parameters in Eq. (1) and the functions in the boundary conditions (2).

When $4b_xd_x < c_x^2$, $4b_yd_y < c_y^2$ and $4b_zd_z < c_z^2$, the closed form solution of Eq. (1) does not exist because the square root of an imaginary unit will be involved. For this case, a numerical method has to be sought to obtain its approximate solution. Since computational efficiency is very important for interactive computer graphics applications, we will concentrate on the analytical solutions.

By altering the position functions, the first derivatives of the boundary conditions (2), and changing the shape parameters, $b$, $c$, $d$ and force function in Eq. (1), Eq. (3) will produce different vase shapes.
3. Vase design by changing functional boundary conditions

The functional boundary conditions not only affect the local features at the boundaries, but also the shape and size of the vase surface. By choosing different functional boundary conditions, vases of different shapes can be easily created. To demonstrate this point, let us have a look at the design of a vase whose opening consists of several pedals and which has a round base.

The boundary conditions can be given as

\[ u = 0 \]

\[ x = R_i \cos a_1 v + R_2 \cos a_2 v + R_3 \cos a_3 v, \]

\[ \frac{\partial x}{\partial u} = R'_i \cos a_1 v + R'_2 \cos a_2 v + R'_3 \cos a_3 v, \]

\[ y = R_i \sin a_1 v + R_2 \cos a_2 v + R_3 \cos a_3 v, \]

\[ \frac{\partial y}{\partial u} = R'_i \sin a_1 v + R'_2 \cos a_2 v + R'_3 \cos a_3 v, \]

\[ z = h_0 + h_1 \sin a_5 v, \quad \frac{\partial z}{\partial u} = h'_0 + h'_1 \sin a_5 v; \]

\[ u = 1 : \]

\[ x = R_4 \sin a_4 v, \quad \frac{\partial x}{\partial u} = R'_4 \sin a_4 v, \]

\[ y = R_4 \sin a_4 v, \quad \frac{\partial y}{\partial u} = R'_4 \sin a_4 v, \]

\[ z = 0, \quad \frac{\partial z}{\partial u} = 0, \quad (7) \]

where \( R_i, R'_i (i = 1, \ldots, 4), h_j, h'_j (j = 0, 1) \) and \( a_k \)

\((k = 1, \ldots, 5)\) are design parameters of the vase. According to the above boundary conditions, \( t_{mi} \)

\((m, i = 1, 2, 3, 4)\) in Eqs. (5) and (6) for the \( x \) component can be determined as follows:

when \( 4b_x d_x < c_x^2 / 4 \)

\[ t_{mi} = \pm a_m \left[ \frac{c_x}{2b_x} \left( 1 + \sqrt{1 - \frac{4b_x d_x}{c_x^2}} \right) \right] \quad (m = 1, \ldots, 4), \quad (8) \]

when \( 4b_x d_x = c_x^2 / 4 \)

\[ t_{mi} = \pm a_m \left( \frac{c_x}{2b_x} \right) \quad (m = 1, \ldots, 4). \quad (9) \]

Similar treatment can be applied to the \( y \) component. For the \( z \) component, only one term that corresponds to \( m = 5 \) exists. In terms of \( B_0(u) \), we have

\[ B_{00}(u) = 0, \]

\[ B_{01}(u) = 0, \]

\[ B_{05}(u) = h_0 + h'_0 u - (3h_0 + 2h'_0)u^2 + (2h_0 + h'_0)u^3. \]

4. Vase design using tangential boundary conditions

Changing the tangential boundary conditions is another effective technique for vase design. This is because the tangential boundary conditions control the rate of change of the surfaces at the boundaries and how fast the effect spreads inwards. Here, we introduce this technique with a simple example.

The boundary curves of the vase, which we are designing, are two circles. Their boundary conditions take the following form:

\[ u = 0 : \]

\[ x = 0.6 \cos v, \quad \frac{\partial x}{\partial u} = r' \cos v, \]

\[ y = 0.6 \sin v, \quad \frac{\partial y}{\partial u} = r' \sin v, \quad (11) \]

Fig. 1. Surface generation by changing functional boundary conditions.

The unknown constants in Eqs. (5) and (6) can be obtained by substituting these closed form solutions into the boundary conditions (7).

To illustrate the effects of the boundary conditions on the designed shape, let us set the design parameters as \( R_1 = 1, R_2 = R_3 = 0.15, \quad R_4 = 0.8, \quad h_0 = 3.6, \quad R'_1 = -2, \quad R'_2 = R'_3 = -0.3, \quad R'_4 = 2, \quad h'_0 = -0.5, h'_1 = 0 \) and the vector-valued shape parameters as \( b_x = b_y = 0.7, c_x = c_y = 2, d_x = d_y = 0.5, b_z = d_z = 1, c_z = 2 \). If we set \( h_1 = -0.2, a_1 = a_4 = 2\pi, a_2 = 8\pi, a_3 = 12\pi, \) and \( a_5 = 10\pi \), the vase surface generated by the closed form solution Eq. (3) of the PDEs is given in Fig. 1a. However, if we make a slight change to these parameters by taking \( h_1 = -0.05, a_1 = a_4 = 2\pi, a_2 = 8\pi, a_3 = 12\pi, \) and \( a_5 = 26\pi \), a new vase surface is produced as shown in Fig. 1b.
where \( r' \) and \( R' \) are the parameters changing the tangential boundary conditions.

In order to examine how the tangential boundary conditions affect the shape of the vase, we keep unchanged the vector-valued shape parameters \( c_x = c_y = 4 \), \( b_x = d_x = b_y = d_y = 1 \) in the closed form solution of Eq. (1) under the boundary conditions (11). Let us only change the tangential boundary conditions, i.e., the first derivatives of Eq. (11). Firstly, we fix the parameter \( r' = -2 \) and see how the variation of \( R' \) affects the surface shape. When choosing \( R' = 1 \), the shape in Fig. 2a was created. When \( R' = 3 \), the shape in Fig. 2b was obtained. The shape in Fig. 2c was from \( R' = -1 \). Then, we fix \( R' = 0 \) and only change \( r' \). When \( r' = -2 \), the image in Fig. 2d was generated. \( r' = 0 \) led to the shape in Fig. 2e and finally, the image in Fig. 2f was produced from \( r' = -3.5 \).

5. Vase design with shape parameters

In this section, we are examining the influence of the vector-valued shape parameters in the proposed fourth order PDEs (1) on the vase shape. In the following design examples, we use the same boundary conditions as Eq. (7), and take the same design parameters as those used in Fig. 1a. If we reset the shape parameters to \( b_x = b_y = b_z = d_x = d_y = d_z = 1 \), \( c_x = c_y = c_z = 2 \) while keeping the other conditions unchanged, the shape of the vase is changed from Fig. 1a to Fig. 3a. Further changing these shape parameters to \( b_x = b_y = b_z = 0.55 \), \( d_x = d_y = d_z = 0.1 \) and \( c_x = c_y = c_z = 2 \), we obtain Fig. 3b. When the parameters are set to \( b_x = b_y = b_z = d_x = d_y = d_z = 1 \) and \( c_x = c_y = c_z = 2 \), the vase shape changes from that shown in Fig. 1b to Fig. 3c. It is apparent from these examples that these shape parameters in the proposed PDEs exert strong influence upon the geometry of the vases generated, and therefore are effective shaping tools for aesthetic designs. It is also clear that there are many combinations of the shape parameters and their effects on the shape of vases are complex. However, in general, if all the components of a vector-valued shape parameter take the same value, the smaller the value of parameter \( b \) or \( c \), or the bigger that of \( d \), the thinner the middle part of a vase will become.

6. Vase design by applying the force function

The force function in Eq. (1) is another important factor affecting the shape of the vases. By applying different force functions or intensity of the applied forces, we can obtain different vase shapes. In general, a positive force pulls the surface outwards, and a negative force pushes the surface inwards. Understandably, the bigger the force, the greater the deformation. Using a simple example, let us discuss the effects of the force function on the vase shapes.

Assume the boundary conditions for this numerical example are as follows:

\[
\begin{align*}
\delta z &= 0, \\
\delta u &= 0, \\
x &= r_0 \cos v, \\
y &= r_0 \sin v, \\
z &= 0,
\end{align*}
\]

Fig. 2. Surface generation by changing tangential boundary conditions.
\[ u = 1 : \]
\[ x = r_1 \cos v, \quad \frac{\partial x}{\partial u} = r_1' \cos v, \]
\[ y = r_1 \sin v, \quad \frac{\partial y}{\partial u} = r_1' \sin v, \]
\[ z = h_1, \quad \frac{\partial z}{\partial u} = h_1'. \] (12)

The force function is taken as
\[ p_x = (p_0 + p_1 u) \cos v, \]
\[ p_y = (p_0 + p_1 u) \sin v, \]
\[ p_z = 0. \] (13)

Due to the existence of the force function, we must seek a particular solution of Eq. (1). Such a particular solution can be obtained by substituting Eq. (13) into Eq. (1), which is given as
\[ x = \frac{1}{d_x} (p_0 + p_1 u) \cos v, \]
\[ y = \frac{1}{d_y} (p_0 + p_1 u) \sin v, \]
\[ z = 0. \] (14)

By taking the design parameters of the boundary conditions (12) as \( r_0 = 0.6, r_1 = 1, h_1 = 3.6, r_0' = 5, r_1' = 1, h_1' = 5, h_1' = 0 \) and the shape parameters as \( b_x = b_y = d_x = d_y = 1, \quad c_x = c_y = 2 \), Fig. 4a shows the vase shape when we choose \( p_0 = 10, \quad p_1 = 1500 \) for the force function. Fig. 4b is obtained when the force function is zeroed, which is the same as the result without the use of the force function.

7. Combined influence

By combining different functional boundary conditions, tangential boundary conditions, shape parameters and force functions, using Eq. (1), we are able to produce vases with even more variety. Here, we consider two groups of combinations, one without the force function (the first group) and one with (the second group).

The boundary conditions used here are the same as Eq. (12). For the first group, we list six combinations in Table 1. They result in a combined effect upon the final design, as illustrated in Fig. 5.

For the second group of designs, let us consider the force function with the form of
\[ p_x = p_0 \sin 3\pi u \cos v, \]
\[ p_y = p_0 \sin 3\pi u \sin v, \]
\[ p_z = 0, \] (15)
which leads to the following particular solution:

\[ x = \frac{p_0}{81 h_x \pi^4 + 9 c_x \pi^2 + d_x} \sin 3\pi u \cos v, \]
\[ y = \frac{p_0}{81 h_y \pi^4 + 9 c_y \pi^2 + d_y} \sin 3\pi u \sin v, \]
\[ z = 0. \]  

From Figs. 5 and 6, it is not hard to see that the combination of different functional boundary conditions, tangential boundary conditions, shape parameters and force functions will allow to produce much more different designs.

The method developed in this paper is computationally very efficient. For all the above examples, it took less than \(10^{-6}\) s to determine the unknown constants of Eq. (3) on an ordinary PC, fast enough for real time interactive surface modelling.

### 8. User interface

In the above we have discussed the theoretical part of the proposed surface modelling method. Using detailed examples, we have also investigated the effects of the shape parameters, boundary conditions and the force functions. Our next step is to develop a user interface, which wraps up all the mathematical formulas and provides the designer with an easy to use design tool. For this purpose, we have designed a prototype system VaseModeler using VC++ and OpenGL. In addition to its use in vase modelling, we hope it will serve as a test bed for verifying the theoretical development.

The resolution engine of VaseModeler is the closed form solution (3)–(6) of the fourth order PDE (1) under the boundary conditions (2). A snapshot of this interface is shown in Fig. 7.

A vase is bounded by its boundary curves. A curve creator is included in VaseModeler for this purpose, from which the user is able to create simple (circles, ellipses, etc.) and complex curves. Then, the curves can be edited through menu entry Edit Curves. For a complicated curve whose mathematical representation does not lead to a closed form solution of PDE (1), its mathematical representation is automatically transformed into Fourier series and therefore can be easily solved with the resolution engine. From menu entry Create Surfaces, simple surfaces (such as spheres and cubes etc.) can be directly created. Complex surfaces can

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be generated from the boundary curves by solving PDE (1). Then, these surfaces can be edited from menu entry Edit Surfaces.

The surface shape is manipulated through menu entry Manipulate Surfaces whose architecture is illustrated in Fig. 8. Users can select the parameters of the positional boundary functions and change their values. The initial tangential boundary conditions are preset by the system according to the mathematical equations of the boundary curves. However, the coefficients of each term or the

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Fig. 6. Surface generation by combining force functions and other parameters.
dominating terms of the Fourier series in the tangential boundary conditions can be interactively changed by the user. All the shape parameters are also given a default value. In Fig. 8 U Term, UV Term and V Term stand for the vector-valued shape parameters corresponding to the first, second and third terms on the left-hand side of PDE (1), respectively. Since these parameters must meet the conditions given in Eqs. (5) and (6), when the value of one shape parameter changed by the user violates these conditions, one of the other two parameters will be automatically adjusted to satisfy the conditions. To help the user compose a sensible line force function or area force function, a number of elementary functions are provided, from which the user can compose a more complex force function using simple arithmetic operators. Once such a force function is constructed, the coefficient of each elementary function can be further modified interactively by the user to adjust the force function to deform the surface shape.

9. Conclusions

Vases represent a special class of free form surfaces (Fig. 9). Although their main bodies are rotational, the important aesthetic elements are the small ornament variations. Conventional surface modelling approaches are not very effective for the generation of such surfaces.
To model vase surfaces easily and efficiently, in this paper, we propose to use a fourth order partial differential equation (PDE) with three vector-valued shape parameters based on our previous work. Considering the boundary conditions, the solution of the proposed PDEs leads to an analytical expression of vase surfaces. Since the proposed PDE has three shape parameters, it is more general and more powerful than the PDEs used for surface generation found in the literature.
Four techniques were introduced for the modelling of vase surfaces, which include the use of functional boundary conditions, tangential boundary conditions, different vector-valued shape parameters and different force functions. To demonstrate the use of the proposed techniques, a number of different vase design samples are presented in this paper.

It is worth pointing out that the existing techniques of PDE based surface generation usually employ a numerical resolution method due to the difficulty of achieving analytical solutions to PDEs. In contrast, our proposed approach emphasises and obtains analytical solutions. This is especially valuable to interactive surface modelling, as computational efficiency is crucially important to such applications. In this paper, we have demonstrated how such an efficient solution can be derived through numerous examples. The use of the prototype interface also demonstrates the user-friendliness of this method.

Although the discussion in this paper is around the application of vase design, the power of the above-proposed PDE based surface modelling method does not stop here. In fact, this method is applicable to many other complex surface design problems, such as blending surfaces [7], compound surfaces with user-specified complex boundary curves and surfaces with splits and creases [11]. The images given in Fig. 10 are such two examples.

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References