Abstract

In this paper, a new mathematical representation of single-patch surfaces is reported. With this representation, complex tool shapes of metal forming can be described with a single surface patch which is prescribed by the boundary of the die at its entry and exit. For non-linearly converging die-surfaces, the die shapes can be effectively altered by changing the coefficients of the tangential boundary conditions. Using the obtained die-surfaces, the finite element analysis of the extrusion of hot metals from a round billet to a hexagon workpiece was carried out. A comparison of the extrusion forces and the grain sizes for the formed parts was made for the linearly and non-linearly converging dies. It was found that the non-linearly converging dies could reduce the deformation resistance remarkably, and produce more uniform grains than those using the linearly converging dies.

Keywords: Tool shape design; Single-patch surfaces; Finite element analysis; Hot extrusion

1. Introduction

Tool shapes have a significant effect on the deformation resistance and the quality of the product. Since tool surfaces may be irregular, they cannot always be represented with simple surfaces such as cylinders and cones. Surface modelling methods have to be employed to produce these shapes. Among them, Bézier, B-spline and NURBS are the most popular, and are widely applied in the design of tool surfaces.

Lee et al. presented a method to design the optimal die profile that can yield more uniform microstructure of a hot extruded product [1]. The die profile of hot extrusion was represented with a Bézier curve. The microstructure evolution of hot extrusion, such as dynamic and static recrystallisation, and grain growth was investigated using Yada and Senuma’s empirical equations and rigid-thermoviscoplastic FEM. Taking a uniform microstructure distribution as an objective function, and using the finite element analysis and an optimisation technique, Jo et al. attempted to optimise the tool shape for hot extrusion and hot forging processes [2]. Defining extrusion die profiles with Bézier curves, Kim et al. applied a coupled numerical approach of finite element analysis and an optimisation technique to achieve an optimal die profile, which yields a more uniform strain rate distribution in the deformed region for the hot extrusion process of metal matrix composites [3]. Using a Bézier curve given in the MARC finite element program and taking five control points to represent the shape of the lateral side of a V-shaped anvil, Duan and Sheppard developed a parameter optimisation system for metal forming by combining the general non-linear finite element analysis software MARC with the improved constrained variable metric strategy [4]. Employing a regular assembly of rectangular patches to model complex-shaped tool surfaces usually leaves some non-four-sided holes. To fill these irregular holes smoothly, Lin et al. adopted a technique of n-sided Bézier patches and developed a simulation method for simulating superplastic forming process of complex-shaped components [5]. They also presented a combined numerical and analytical method to investigate the error of approximating circular arcs using Bézier curve segments and simulated the processes of superplastically forming a 3D rectangular box with fillet surfaces and wrapping a decorative pattern onto an axisymmetric ceramic pot [6]. Using a cubic Bézier curve to describe a streamline, Chitkara and Celik proposed an analytical method based on the upper-bound theory to investigate the 3D off-centric extrusion of arbitrarily shaped sections from
Gunasekera [15] presented a new upper-bound solution for extrusion of circular sections from initially circular billets using both linear and smooth curved dies [8]. In addition to Bézier curves and surfaces, B-spline and non-uniform rational B-spline (NURBS) were also employed to model tool surfaces.

Shimizu and Sano used B-spline patches to model tool surfaces and developed a penalty method for the analysis of contact and friction of metal forming using the finite element method [9]. Representing the preform die shapes with cubic B-spline curves and using the control points or coefficients of B-spline as design variables, Zhao et al. discussed how to minimise the zone where the realised and desired final forging shapes do not coincide in the net-shape forging process [10].

Since the use of NURBS can model multiple complex surfaces with a single mathematical description, Sadeghi and Pursell studied the rigid contact surfaces of superplastic forming with complex die shapes and performed finite element modelling [11]. Shim and Suh proposed a contact treatment algorithm for trimmed NURBS surfaces and applied it to the deep-drawing of a clever-shaped cup and an L-shaped cup [12].

Extrusion forming is one of the most common metal forming methods. However, research is mainly focused on linearly converging tools. For example, Boer et al. presented a theoretical approach for the analysis of the direct drawing of square section rod with various corner radii from a round bar [13]. Gunasekera and Hoshino [14] and Hoshino and Gunasekera [15] presented a new upper-bound solution for the extrusion of square sections from round billets through converging and curved dies. Boer and Webster compared the results of upper-bound and FEM using the drawing of a square section from a round billet and discussed both the advantages and disadvantages of these methods [16]. Saahou et al. [17] modified the spatial elementary rigid region (SERR) technique proposed by Gatto and Giarda [18] and reformulated by Kar and Das [19]. Gouveia et al. analysed the 3D forward extrusion of a square section from a round billet through a straight converging die using both the updated Lagrangian and the combined Eulerian–Lagrangian finite element formulations [20]. They also investigated 3D forward extrusion from a round billet to a square bar through a linear die using both physical modelling and numerical simulations [21].

For these forming processes, the tool surfaces are non-axisymmetric and the whole tool surfaces cannot be represented with a single Bézier, B-spline or NURBS patch. If multiple patches are employed, many variables are involved which make the tool shape design and analysis processes more complicated. In particular, if finite element analysis is required, the design and analysis processes will become much more time-consuming than for single-patch surfaces. Furthermore, in order to extend the analysis and design from linearly to non-linearly converging non-axisymmetric extrusion dies, it is necessary to find an effective method for representing the surfaces of non-linearly converging dies. Therefore, developing a new method to represent the shape of tool surfaces and using only a small number of design variables for tool surface control will be very useful for the design of tool surface shapes.

In this paper, the authors propose to use single-patch surfaces to represent complex shapes. By constructing a suitable function from the boundary conditions of the entry and exit of the tool surfaces, both linearly and non-linearly varying tool surfaces can be generated with the function. For non-linearly varying tool surfaces, the surface shape can be effectively changed by two tangential boundary conditions. In order to demonstrate the advantages of the non-linearly converging tool surfaces over the linearly converging tool surfaces, finite element simulations are carried out for the extrusion forming of a hexagon rod from a round bar using both linear and non-linear dies.

2. Single-patch surfaces

As discussed above, the traditional geometric modelling methods such as Bézier, B-spline and NURBS are not only commonly applied in computer aided design and computer graphics, but also used widely in tool shape design of metal forming. In addition to these common geometric modelling methods, another modelling method is attracting the attention of the geometric modelling community. This method is based on the solution to a suitably chosen partial differential equation (PDE). It was firstly proposed by Bloor and Wilson [22]. In their work, a fourth-order partial differential equation with a vector-valued parameter was employed.

Applying the theories of elasticity, a fourth-order partial differential equation can be derived from the deformation of an elastic plate. It indicates the coefficients in the fourth-order partial differential equation are relevant to the physical and mechanical properties of a plate. Therefore, PDE-based modelling method can be regarded as physics-based.

By suitably constructing the surface functions, the PDE-based geometric modelling method can represent some complex surfaces with a single patch. Many conventional geometric modelling methods deform curved surfaces through control points. Therefore surface manipulation can be tedious if the number of the control points is large. PDE surfaces on the other hand can be manipulated by changing the vector-valued parameters, the force function and the boundary conditions of the PDE. Zhang and You have discussed how these factors affect the shapes of the generated surfaces [23].

One main problem using the solution of a partial differential equation to generate surfaces is how to solve the equation efficiently. The numerical methods, although effective, are computationally expensive. Although the closed-form
solution is efficient, it is only applicable to some simple surface generation problems. The Fourier series method and the piecewise method proposed by Bloor and Wilson are also efficient methods [24]. Unfortunately, both are inaccurate and hence are unsuitable for the representation of forming tools where accuracy is paramount.

In order to represent tool surfaces with a single patch, the authors have proposed a truncated power series for the design of tool surfaces [25]. The finite element simulation of a round-to-square extrusion of hot metals has also been carried out. In this paper, the authors extend their previous work and develop a new method for the modelling of forming tool surfaces.

A tool surface usually has two boundaries: the entry boundary and the exit boundary. Using a parametric description where the parametric variable \( u \) is along the axis of the tool surface and the parametric variable \( v \) is along the circumferential direction, the boundary conditions at these boundaries can be represented as the functions of the parametric variable \( v \). For a tool surface which varies linearly along the \( u \)-direction, the boundary conditions at the entry and exit can be written as

\[
u = 0: \quad x = G_1(v), \quad u = 1: \quad x = G_2(v)
\]

where \( x(u, v) = [x(u, v), y(u, v), z(u, v)]^T \) and \( G_i(v) = [G_{1i}(v), G_{2i}(v), G_{3i}(v)] \) \( i = 1, 2 \).

It can be seen that for such tools, there is no tangential continuity at the entry and exit. Therefore, the flow direction of the forming metals will change at these two positions.

According to the boundary conditions (1), a function of the tool surfaces can be assumed as follows:

\[
x = f_1(v) + u f_2(v)
\]

Substituting Eq. (2) into Eq. (1), the unknown functions \( f_1(v) \) and \( f_2(v) \) are determined, and have the forms of

\[
f_1(v) = G_1(v), \quad f_2(v) = G_2(v) - G_1(v)
\]

With Eq. (3), the surface function (2) of tools becomes

\[
x = (1 - u)G_1(v) + uG_2(v)
\]

It can be observed from Eq. (4) that surface \( x \) is a linear function of the parametric variable \( u \). The tools designed with such a function are called linearly converging tools.

In order to ensure the tangential continuity of the metal flow at the entry and exit, two additional tangential conditions must be introduced and the boundary conditions (1) are changed to

\[
u = 0: \quad x = G_1(v), \quad \frac{\partial x}{\partial u} = G_2(v), \quad u = 1: \quad x = G_2(v), \quad \frac{\partial x}{\partial u} = G_1(v)
\]

where \( G_i(v) = [G_{1i}(v), G_{2i}(v), G_{3i}(v)] \) \( i = 1, 2, 3, 4 \). With the boundary conditions (5), it can be assumed that the tool surfaces have the following forms:

\[
x = f_1(v) + u f_2(v) + u^2 f_1(v) + u^3 f_1(v)
\]

Similarly, substituting Eq. (6) into the boundary conditions (5), the unknown functions of Eq. (6) can be determined as follows:

\[
\begin{align*}
f_1(v) &= G_1(v), \\
f_2(v) &= G_2(v), \\
f_3(v) &= -3G_1(v) - 2G_2(v) + 3G_3(v) - G_4(v), \\
f_4(v) &= 2G_1(v) + G_2(v) - 2G_3(v) + G_4(v)
\end{align*}
\]

Substituting Eq. (7) into Eq. (6), the mathematical representation of the tool surfaces are written as

\[
x = (1 - 3u^2 + 2u^3)G_1(v) + (u - 2u^2 + u^3)G_2(v) + (3u^2 - 2u^3)G_3(v) + (-u^2 + u^3)G_4(v)
\]

Since the surface function \( x \) determined by Eq. (8) is a non-linear function of the parametric variable \( u \), the tools generated are called non-linearly converging tools.

3. Transformation of complicated boundary conditions

In the boundary conditions (1) and (5), every boundary curve for linearly converging tools or every boundary curve and boundary tangent for non-linearly converging tools are described with only one mathematical equation. However, one mathematical equation is not adequate for some complicated boundary curves. For these cases, the boundary conditions must be transformed into the forms of a Fourier series. For example, for a round-to-hexagon extrusion, the boundary curve of the extrusion die at the entry is a circle having the mathematical equations in parametric forms:

\[
x = r \cos 2 \pi v, \quad y = r \sin 2 \pi v
\]

where \( r \) is the radius of the boundary circle at the entry of the die.

However, the boundary curve at the exit is a hexagon which is described with the following sets of seven equations:

\[
x = \frac{1}{2} \sqrt{3}a, \quad y = 6av, \quad 0 \leq v \leq \frac{1}{12};
\]

\[
x = 3\sqrt{3}(\frac{1}{2} - v), \quad y = a(\frac{1}{2} + 3v), \quad \frac{1}{12} \leq v \leq \frac{1}{6};
\]

\[
x = 3\sqrt{3}(\frac{1}{2} - v), \quad y = a(\frac{1}{2} - 3v), \quad \frac{1}{6} \leq v \leq \frac{5}{12};
\]

\[
x = -\frac{1}{2} \sqrt{3}a, \quad y = 3a(1 - 2v), \quad \frac{5}{12} \leq v \leq \frac{7}{12};
\]

\[
x = -\frac{3}{2} \sqrt{3}a(\frac{1}{2} - v), \quad y = a(\frac{1}{2} - 3v), \quad \frac{7}{12} \leq v \leq \frac{1}{2};
\]

\[
x = -3\sqrt{3}(\frac{1}{2} - v), \quad y = -a(\frac{1}{2} - 3v), \quad \frac{1}{2} \leq v \leq \frac{11}{12};
\]

\[
x = 3\sqrt{3}(\frac{1}{2} - v), \quad y = -6a(1 - v), \quad \frac{11}{12} \leq v \leq 1
\]

where \( a \) denotes the diagonal length of the hexagon.

In order to use a mathematical equation to describe the hexagon, a Fourier series is applied which has the forms of
where \( a_{in} \) \((i = x, y, n = 1, 2, \ldots, N)\) and \( b_{in} \) \((i = x, y, n = 1, 2, \ldots, N)\) are the Fourier coefficients.

Using the curve-fitting technique for the \( x \) and \( y \) components, all Fourier coefficients in Eq. (11) can be determined.

The entry and exit of the die are assumed to be at \( z = 0 \) and \( z = H \), respectively, which means the length of the die is \( H \). For linearly converging extrusion dies, the boundary conditions only contain the information of boundary curves and can be written as follows:

\[
u = 0: \quad x = r \cos 2\pi v, \quad y = r \sin 2\pi v, \quad z = 0 ;
\]

\[
u = 1: \quad x = a_{00} + \sum_{n=1}^{N} a_n \cos n\pi v + b_n \sin n\pi v, \quad y = a_{00} + \sum_{n=1}^{N} b_n \cos n\pi v + a_n \sin n\pi v, \quad z = H
\]

According to Eq. (4) and the boundary conditions (12), the surface functions of the linearly converging dies are

\[
x = (1 - u)r \cos 2\pi v + a_{00} + \sum_{n=1}^{N} a_n \cos n\pi v + b_n \sin n\pi v ,
\]

\[
y = (1 - u)r \sin 2\pi v + a_{00} + \sum_{n=1}^{N} b_n \cos n\pi v + a_n \sin n\pi v, \quad z = Hu
\]

Taking the first derivatives of \( x \) and \( y \) components with respect to the parametric variable \( v \) to be zero and those of \( z \) component to be \( \eta_1 \) at the entry and \( \eta_2 \) at the exit, the boundary conditions defining the non-linearly converging dies can be written as

\[
u = 0: \quad x = r \cos 2\pi v, \quad \frac{\partial x}{\partial \nu} = 0, \quad y = r \sin 2\pi v, \quad \frac{\partial y}{\partial \nu} = 0, \quad z = 0 , \quad \frac{\partial z}{\partial \nu} = \eta_1;
\]

\[
u = 1: \quad x = a_{00} + \sum_{n=1}^{N} a_n \cos n\pi v + b_n \sin n\pi v, \quad \frac{\partial x}{\partial \nu} = 0, \quad y = a_{00} + \sum_{n=1}^{N} b_n \cos n\pi v + a_n \sin n\pi v, \quad \frac{\partial y}{\partial \nu} = 0, \quad z = H , \quad \frac{\partial z}{\partial \nu} = \eta_2
\]

According to Eq. (8) and the above boundary conditions (14), the surface functions of the non-linearly converging dies become

\[
x = (1 - 3u^2 + 2u^3)r \cos 2\pi v + (3u^2 - 2u^3) \left[ a_{00} + \sum_{n=1}^{N} a_n \cos n\pi v + b_n \sin n\pi v \right],
\]

\[
y = (1 - 3u^2 + 2u^3)r \sin 2\pi v + (3u^2 - 2u^3) \left[ a_{00} + \sum_{n=1}^{N} b_n \cos n\pi v + a_n \sin n\pi v \right],
\]

\[
z = (u - 2u^2 + u^3)\eta_1 + (3u^2 - 2u^3)H + (-u^2 + u)\eta_2
\]

Taking \( r = 150 \) mm and \( H = 300 \) mm, and using Eq. (13), the authors generate the surface of a linearly converging die as depicted in Fig. 1(a). The surface of a non-linearly converging die is created using Eq. (15). When taking \( \eta_1 = \frac{H}{2} = 1 \), the surface of the non-linearly converging die is obtained as shown in Fig. 1(b). The surface in Fig. 1(c) and (d) are from \( \eta_1 = \eta_2 = 3 \) and \( \eta_1 = \eta_2 = 6 \), respectively. From these images, it is clear that the surface shapes of the tool surface can be effectively changed by employing different values of \( \eta_1 \) and \( \eta_2 \). In addition, it can be observed that the linearly converging die (Fig. 1(a)) cannot ensure the tangential continuity of the die along the \( z \)-direction.
whereas the non-linearly converging die in Fig. 1(b) and (c) satisfies tangential continuity along the z-direction.

4. Finite element analysis of round-to-hexagon extrusion

The above-obtained linearly and non-linearly converging extrusion dies are used for the finite element analysis of the extrusion forming of a hexagon workpiece from a round bar. Since the cross-section of the workpiece is a hexagon, the symmetrical part is one-twelfth of the section. Here the authors take one quarter of the section for the finite element analysis. A total of 21 nodes are uniformly collocated along the circumferential direction, 21 nodes along the radial direction, and 21 nodes along the axial direction of the extrusion die. The die and the punch are treated as rigid shell elements. The eight-node 3D solid elements are used to represent the workpiece. The finite element model is shown in Fig. 2. The exit of the die is at Z = 200 mm.

The diameter and the height of the workpiece are 300 mm, and the height of the die is 600 mm. The punch moves along the z-direction beginning from the location where it contacts the workpiece. In order to reduce the amount of the finite element computation, it is assumed that the punch can move to the exit of the die. Therefore, the maximum displacement of the punch is $S = 600$ mm.

The friction at the workpiece-tool interface is taken into account by the Coulomb friction law. An exponential function is used to smoothly interpolate the static $\mu_s$ and dynamic $\mu_d$ friction coefficients, i.e.

$$\mu = \mu_d + (\mu_s - \mu_d) e^{-v/c} \quad (16)$$

where $v$ is the relative velocity between the slave node and the master segment and $c = 3$ is the decay constant, $\mu_s = 0.3$ and $\mu_d = 0.1$.

Strain-rate-dependent material models are widely applied in analysis of the hot metal forming. Here a piece-wise linear isotropic plasticity constitutive relationship [26] is used which includes the Cowper–Symonds multiplier [27] to account for the strain rate:

$$\bar{\sigma} = \left[1 + \left(\frac{\dot{\varepsilon}}{\eta}\right)^{1/P_s}\right] \left[\eta_0 + E_p\dot{\varepsilon}^d\right] \quad (17)$$

where $\bar{\sigma}$ is the yield stress, $\eta_0 = 22$ MPa the initial yield stress, $\dot{\varepsilon}$ the strain rate, $C = 4000$ s$^{-1}$ and $P = 5$ the Cowper–Symonds strain rate parameters, $E_p = 18$ MPa the plastic hardening modulus, and $\dot{\varepsilon}^d$ is the effective plastic strain.

Comparison of the extrusion forces of the linear and the non-linear dies

<table>
<thead>
<tr>
<th>Dies</th>
<th>$F$ (tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>514.4</td>
</tr>
<tr>
<td>Non-0</td>
<td>514.2</td>
</tr>
<tr>
<td>Non-1</td>
<td>449.4</td>
</tr>
<tr>
<td>Non-2</td>
<td>426.9</td>
</tr>
<tr>
<td>Non-3</td>
<td>417.6</td>
</tr>
<tr>
<td>Non-4</td>
<td>441.1</td>
</tr>
<tr>
<td>Non-6</td>
<td>500.9</td>
</tr>
</tbody>
</table>

Non0-6 represent the non-linear tools with tangential values varying from 0 to 6, respectively.

Non-0-6 represents the non-linearly converging die.

Fig. 2. The finite element model.
Fig. 3. Comparison of the extrusion forces between the linear die and the non-linear die with tangential value $\eta = 3$.

\[
\delta(\%) = \frac{|F_{\text{non-linear}} - F_{\text{linear}}|}{F_{\text{non-linear}}} = \frac{|417.6 - 514.4|}{417.6} \times 100 = 23.18 \quad (18)
\]

This suggests that the non-linear die with tangential value $\eta_1 = \eta_2 = 3$ can reduce the deformation resistance remarkably compared to the linear counterpart.

Figs. 4 and 5 are the plastic strain contours of the formed workpiece when the punch moves to $S = 600$ mm. Fig. 4 corresponds to the linear die given in Figs. 1(a) and 5 to the non-linear die given in Fig. 1(c). The effective plastic strain for the linear die varies from $7.232 \times 10^{-1}$ to $3.679$ and that for the non-linear die varies from $7.954 \times 10^{-1}$ to $3.34$. Clearly, the non-linear die gives a smaller variation of the effective plastic strain than the linear die.

Figs. 6 and 7 describe the plastic strain rate and the plastic strain contours of the non-linear model at section $Z = 200$ mm (the outlet of the die) and $S = 300$ mm. Figs. 8...
Fig. 8. The plastic strain rate contour of section $Z = 200\, \text{mm}$ and $S = 300\, \text{mm}$ for the linear die.

and 9 describe the plastic strain rate and the plastic strain contours of the linear model at section $Z = 200\, \text{mm}$ (the outlet of the die) and $S = 300\, \text{mm}$.

From Figs. 7 and 9, it can be seen that the effective plastic strain of the non-linear die varies less than that of the linear die.

Figs. 6 and 8 show that the effective plastic strain rate of the non-linear die varies from $7.5 \times 10^{-5}$ to 0.05, and that of the linear die varies from 6.8 $\times 10^{-5}$ to 0.14. Therefore the non-linear die also gives a smaller variation of the effective plastic strain rate than that of the linear die.

Using the equations below, the relationship between the grain size and the thermomechanical parameters can be determined:

$$d = 87^{-1}d_0^{0.48}Z^{-0.16} \quad \text{(19)}$$

where the Zener–Holloman parameter $Z$ is

$$Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) \quad \text{(20)}$$

and $Q, R, T, d_0, \dot{\varepsilon}$ and $\dot{\varepsilon}$ are the activation energy, gas constant, absolute temperature, initial grain size, effective plastic strain and effective plastic strain rate, respectively.

These equations suggest that the grain size depends on the effective plastic strain and effective plastic strain rate. The more they vary, the greater is the variation of the grain size. Since the non-linear dies have a smaller variation of the effective plastic strain and effective plastic strain rate, the variation of the grain size given by the non-linear dies is smaller than that given by the linear dies.

5. Conclusions

A method to model single-patch surfaces has been presented in this paper. It is based on the proper construction of a surface function defined by the boundary conditions. With such a method, complex tool shapes for both linearly and non-linearly converging tools can be represented with one single surface patch. Furthermore, the shapes of the non-linearly converging tools can be effectively altered with the boundary tangent values.

By transforming complex boundary curves which cannot be represented with one mathematical equation into Fourier series, the tool surfaces defined with such boundary curves and boundary tangents can also be modelled with one surface patch.

As an application of the proposed method in metal forming, both linearly and non-linearly converging extrusion dies have been generated and how the tangential boundary conditions affect the tool shapes has been investigated. Using the software package LSDYNA, the finite element analysis of the extrusion forming of a hexagonal workpiece from a round bar has been performed. It is found that the non-linearly converging dies can reduce the extrusion force greatly, and produce more uniform grains than the linearly converging dies.

References


