Cellular Formation and Representation

Joshua Bainbridge
MSc Computer Animation and Visual Effects,
Bournemouth University

November 21, 2014

Abstract

Within this report the author will review modern developments in cellular formation as a result of simulation. This will also involve the representation of such data and the potential advancements of working within homogeneous systems.

1 Introduction

Synthetic representation of organic life has been a continual focus within the field of computer graphics since its inception. The issue of recreating organic information is that the process of doing so is incredibly complex and as a result computationally expensive. Another issue is that accurate calculation when representing such data increases proportionally to the datas complexity. In recent years however, with the increase in computational power and the development of parallel computation, such simulations and accurate representation has become more feasible and realistic. Algorithms have also been developed that can accelerate this process and be utilized in a homogeneous sense with both simulation and representation.

2 Related Work

There has been much development in organic simulation as well as representation using light transportation, as a result we will address each subject separately with a level of assumed knowledge.

2.1 Simulation

The process of deriving organic structures can be categorized into either that of addition or multiplication. In 1981, Witten and Sander proposed a simple theory of growth by repeated deposition in an article named "Diffusion-Limited Aggregation, a Kinetic Critical Phenomenon" [9]. This simple Monte Carlo algorithm of deposition and addition to a larger aggregate forms organic like

structures of fractal space. It was discovered as a result of one of Witten's students recording repeated formations of colloidal iron as a result of thermal Brownian motion. Andy Lomas used Witten and Sander's algorithm in his sketch "Aggregation: Complexity out of Simplicity" [4] for the purposes of artistic exploration within implicit, mathematical concepts of organic processes.

Lomas then later published a paper called "Cellular Forms: An Artistic Exploration of Morphogenesis" [5] in which he describes an algorithm for simulating a simplification of the morphogenetic process. This is instead a form of multiplication as it does not use deposition but rather the splitting of cells across a linked structure forming more complex forms over time. The parameters of the morphogenetic process are governed by levels of nutrients within each cell and the diffusion of such nutrients across the structure. Controlling the diffusion of nutrients is not necessarily uniform and Lomas uses two-component reactiondiffusion equations originally proposed by Alan Turing in his paper "The Chemical Basis of Morphogenesis" [7]. These differential equations represent the evolution of the chemical substances in the form:

$$\frac{dA}{dt} = K\nabla^2 A + P(A, B)$$

Where A and B are fields that represent the concentration of chemical substances, K is the diffusion coefficient and $P(A, B)$ accounts for local reactions. Then there would be an equivalent equation for the B field so that each field can then be represented in uv coordinates. The resulting two dimensional gradients form organic patterns often seen in nature.

2.2 Representation

As technology advances and our capacity for more complex computation increases there has been much development in numerical solutions to light transportation. A paper called The Rendering Equation [2] was first published in 1986 by James Kajiya. In this paper, Kajiya defined the integral equation for physically plausible light transportation and the means to solve it using Monte Carlo estimation. It involved an accurate distribution of energy along a path of light traced from the camera back to the source.

This brute force approach however was slow to converge as small lights would have a low probability of ray intersection resulting in sample variance. Eric Lafortune and Yves Willems in 1993 proposed a new technique in their paper Bi-Directional Path Tracing [3]. This involved tracing the ray from sources of light within the scene as well as from the camera. Each intersection from a ray originating at the camera could then sample that of the lights intersections, gathering illumination indirectly. This would increased performance in relation to small light sources as well as caustics. It was then further developed by Eric Veach and Leonidas Guibas in their paper Optimally Combining Sampling Techniques for Monte Carlo Rendering [8] in which they proposed a technique called multiple importance sampling. This meant that multiple sampling methods

could be evaluated, weighting the probability between each method resulting in variance reduction.

3 Cellular Formation

In Lomas's paper [5] he describes a mathematical model that performs a simplified process similar to that of morphogenesis. The starting axiom for such a model is a surface of interlinking cells. Measuring the distance to each linked cell and finding the average length dictates the cells radius. Multiple influences are then applied to each cell creating target vectors. These influences are as a result of the linked and surrounding cells applying forces upon each other creating form. To apply these forces, normal vectors for each cell on the surface must be found. Calculating this means averaging the normal from each triangle formed of L_n and L_{n+1} where L_n represents a linked cell. As a result it is important that links of each cell be in a specific order and that the rotational direction of the links be in accordance to all other cells, producing normals facing outward from the surface.

The first of the these influences is the *springTarget* force. This is a linear spring like system where each link pushes the cell to maintain a rest position between the cells. The position found from each links influence is then averaged to get the final target position from the spring simulation. In Lomass paper it is written in this form:

$$springTarget = P + 1/n \sum_{r=1}^{n} (linkRestLength(\widehat{L_r - P}))$$

Where P represents the current position and L_r is the position of the linked cells while *linkRestLength* is a parameter of the simulation. However the use of P outside of the summation introduces a constant form of addition to the *linkRestLength* giving an accumulative result which is undesired. Here we propose this as an alternative:

$$springTarget = 1/n \sum_{r=1}^{n} (L_r + linkRestLength(\widehat{P - L_r}))$$

Here the direction of the vector is now from the link position towards the cell maintaining a constant distance. This is then averaged as before between all the individual link targets. The next influence is the *planarTarget* which averages the link positions, smoothing the surface of the structure. It can be written as:

$$planarTarget = 1/n \sum_{r=1}^{n} L_r$$

This influence when used separately from the other forces will reduce the structure to a single point but when used in conjunction will spread the cells across the surface. The third influence upon each cell is the *bulgeTarget*. It will push the

cell along its current normal direction until the distance between the current cell and the link has reached the link rest length. As with the *springTarget*, the resulting target positions from each link individually are then averaged to produce the final target position from the bulge simulation. It can be written in the form:

$$dotN_r = (L_r) \cdot N$$
$$bulgeDist = 1/n \sum_{r=1}^{n} (\sqrt{linkRestLength^2 - |L_r|^2 + {dotN_r}^2} + dotN_r)$$
$$bulgeTarget = P + bulgeDist \cdot N$$

Where N is the unit normal of the current cell. This has the effect of resisting against the influence of $planarTarget$, causing the surface of the structure to bulge outward and away from the linked cells. The vectors between the current cell and the targets produced by each influence are found and weighted according to parameters of the simulation. These weighted vectors are then added to each corresponding cell in the structure.

Lomas also describes another influence of repulsion between cells that imposes constraint and structural coherency. It represents how physics imposes constraints on organic structures and on how they grow. This influence is called *collisionOffset* and takes into account all cells within a radius. It also excludes any cells linked to the current cell as their influence has already been accounted for. It is defined as:

$$collisionOffset = repulsionStrength \sum_{r \epsilon A} (\frac{roi^2 - |P - P_r|^2}{roi^2} (\widehat{P - P_r}))$$

Where $repulsionStrength$ is the amount of influence, roi is the radius of influence and P_r represents each cell within that radius. When the distance between the influencing and current cells near the value of roi, the less it will contribute to due to exponential falloff. This value is then added to the current cell's position.

During the simulation, Lomas adds nutrients to each cell. This accumulation of nutrients can either be constant or uniformly random across the surface. The nutrients can then be diffused along the links between cells. Lomas also experimented with Turings two-component reactiondiffusion equations. The model proposed by Turing is that of morphogenesis on a chemical level representing two independent species or substances. The visual effect of Turing's work often resembles complex patterns found in nature. When using these equations for the purposes of diffusion, Lomas found little added complexity to the structures form. He did however propose that with further development these equations could be used to control simulation parameters across the surface, giving variation to the types of cell. This accumulation of nutrients is then used to trigger cell division creating a daughter cell. This process involves the parent cell randomly finding two bisecting links and removing all links on one side. The daughter cell is then created and linked to each disconnected cell, the parent and the corresponding cells to the bisecting links. The daughter cells initial position is the the average

of all linked cells. As normals will be needed for calculating influences during the simulation it is important to also take into account the order of linked cells. This is not only limited to that of the daughter cell but also the parent and all other linked cells as their relationships are being mutated.

This process is then repeated creating larger cell structures over time at an exponential rate. The characteristics of the structure then become more visually evident. These characteristics are defined by the simulation parameters.

4 Rendering

Representing such information produced with the techniques proposed by Lomas is another issue. Kajiya [2] originally defined the rendering equation that took into account the physical properties of light and its transportation and distribution between surfaces. It can be written in this form:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega f_r(x, \omega_i, \omega_o) L_i(x, \omega_i)(\omega_i \cdot n) d\omega_i$$

Where $L_o(x, \omega_o)$ is the outgoing light at x in direction ω_o, $L_e(x, \omega_o)$ is the light being emitted from the surface and $L_i(x, \omega_i)$ is the incoming light from across the hemisphere. This $(\omega_i \cdot n)$ is called the cosine law and represents the dot product of the incident light direction and the normal direction of the surface. That is the differential area of light being projected onto the base of the hemisphere. As the angle between the two vectors increase the projected area also increases resulting in the energy being distributed. The $f_r(x, \omega_i, \omega_o)$ represents a bidirectional scattering and distribution function. This returns the probability of the light coming from ω_i and reflecting in the direction of ω_o. If you were to integrate across the entire hemisphere this would balance itself to a value of one.

In Kajiyas paper [2] he also proposes a numerical solution to this integral equation using monte carlo techniques. The problem is that to converge on an accurate estimation of the integral with a brute force approach it takes $O(n^2)$ time. This can be improved however with more advanced sampling to reduce variance. To do this Kajiya proposed a technique called importance sampling where the direction of ω_i is weighted towards the bidirectional scattering and distribution function. This is so that the contribution of each new sample will have a higher probability of contributing to the final value. As samples are also attenuated each time they bounce as a result of energy being absorbed, their contribution to the final image becomes less. This can also be optimised with russian roulette [6] to control ray depth termination giving an estimation. This usually reduces variance although with incorrect parameters can also increase it.

In Veachs paper [8] he describes another approach in which not only the bidirectional scattering and distribution function is sampled but also the lights probability. The probabilities are then weighted reducing variance. The advantages of this technique becomes more apparent when smaller light sources are used in conjunction with bidirectional scattering and distribution functions that have an even distribution. If the lights themselves are then also sampled,

the cosine rule must be applied to the lights surface as well as an exponential decrease in energy over distance to account for distribution. An exponential value can also be introduced to the cosine rule to give a weighted direction to the light being emitted from a planar surface.

Indirect paths to light sources can still be a source of variance but this can be further reduced with the use of bi-directional path tracing as proposed by Lafortune and Willems [3]. This approach works in the same way as standard path tracing but also traces paths from the light itself. The intersections found as a result of paths originating from the lights are then accounted for in each intersection of the camera rays.

Properties of the camera can also be taken into account such as depth of field and motion blur as described by Robert Cook [1]. These however are also randomly sampled and will contribute to variance within the image. Depth of field can be represented using geometric optics and the definition of an aperture. The aperture is positioned at the nodal plane and is randomly sampled within a predefined radius. This defines the sample position with a direction towards an intersection of the focal plane and a ray starting at the film plane and travelling through the nodal point. Motion blur can be taken into account by randomly finding a delta time value and interpolating the camera and geometric data for each sample.

5 Integration

In section three of this report we reviewed techniques for cellular growth, in section four we looked at radiometric transportation. These two subjects, simulation and representation are distinctly different however can be integrated to achieve interesting results. Lomas uses raytracing techniques within his simulation for introducing nutrients to the cellular formation. As a result the formation grows in the direction of the radiating rays. This responsive growth is similar to that of plants to enable the process of photosynthesis. However this technique could be taken further using more accurate simulations of light such as that described by Kajiya. This with more advanced models to simulate nutrient diffusion throughout the structure could produce further developments in synthetic plant simulation. Acceleration algorithms can also be shared across both subjects as they deal with issues of spatial representation. Unlike hierarchical structures that would be inefficient with a large group of cells working independently, spatial acceleration groups cells according to position allowing for faster neighbour lookup. The added benefit is that this same acceleration structure can be used for ray intersections. This becomes more apparent over time as the computational complexity grows at a rate of $O(2^n)$ which is compounded when calculating *collisionOffset* at $O(n^2)$.

6 Conclusion

We can see that with current techniques and computational hardware, organic simulation and representation despite its inherent complexity is a constant developing area of research. Simulation models are becoming more accurate at representing real life phenomenon, relating well to the industries movement towards realism in animation and film.

References

[1] Robert L. Cook, Thomas Porter, and Loren Carpenter. Distributed ray tracing. In *Proceedings of the 11th Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '84, pages 137–145, New York, NY, USA, 1984. ACM.

[2] James T. Kajiya. The rendering equation. In *Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '86, pages 143–150, New York, NY, USA, 1986. ACM.

[3] Eric P. Lafortune and Yves Willems. Bi-directional path tracing. In *Compugraphics '93, Compugraphics '93, Alvor, Portugal*, pages 145–153, Dec 993.

[4] Andy Lomas. Aggregation: Complexity out of simplicity. In *ACM SIGGRAPH 2005 Sketches*, SIGGRAPH '05, New York, NY, USA, 2005. ACM.

[5] Andy Lomas. Cellular forms: An artistic exploration of morphogenesis. In *ACM SIGGRAPH 2014 Studio*, SIGGRAPH '14, pages 1:1–1:1, New York, NY, USA, 2014. ACM.

[6] Lszl Szcsi, Lszl Szirmay-Kalos, and Csaba Kelemen. Variance reduction for russian-roulette. In *WSCG*, 2003.

[7] Alan Turing. The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society B*, 237:37–72, 1952.

[8] Eric Veach and Leonidas J. Guibas. Optimally combining sampling techniques for monte carlo rendering. In *Proceedings of the 22Nd Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH '95, pages 419–428, New York, NY, USA, 1995. ACM.

[9] T. A. Witten and L. M. Sander. Diffusion-limited aggregation, a kinetic critical phenomenon. *Phys. Rev. Lett.*, 47:1400–1403, Nov 1981.