Effects of different order PDEs on blending surfaces

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Abstract

In this paper, we introduce second order and mixed order partial differential equations (PDEs) for surface blending and present an approximate algorithm for the resolution of the PDEs. We investigate how different orders of partial differential equations influence the continuities of blending surfaces.

Keywords---surface blending, different orders of partial differential equations, effects on blending surfaces

1. Introduction

Surface blending has a wide range of applications in computer aided design, computer animation and other industries. It has attracted a lot of research attention.

Rossignac and Requicha defined the rolling-ball blending method in terms of offset solids and approximated blending surfaces and their intersecting curves by use of piecewise circular curves [1]. Middledich and Sears proposed a technique which is suitable for blend definition and profile control. With such a technique, determining the blending surface with this method relies on finding a conic tangential to two straight lines that define a plane [2]. Hoffmann and Hopcroft proposed the formulation for blending two or three primary surfaces and named this technique the potential method [3]. Later on, they developed the projective potential method [4]. Bloor and Wilson introduced the fourth order partial differential equation into surface blending [5]. Rockwood proposed a method to blend implicit surfaces and called it the displacement method [6]. For implicit surfaces, Warren introduced a new definition of geometric continuity and presented some new methods to blend several primary surfaces simultaneously [7]. In addition to the application in blending implicit surfaces, the rolling ball method also finds a wide range of applications in blending parametric surfaces. Normally, there are two kinds of rolling-ball blends: constant-radius blends and variable-radius blends. By sweeping rational quadratic (conic section) curves, Choi and Ju constructed constant-radius rolling-ball (edge) blends which can be used to blend any rectangular parametric surface patches with smooth offset surfaces whose intersections can be well defined [8]. Later, Kosters extended the potential method to blend implicitly defined surfaces, includes blending an arbitrary number of surfaces to form a convex corner, blending surfaces for all cases where three surfaces meet transversely, and blending for a very general class of corners [9]. He further extended the potential method to blends of arbitrary continuities consisting of piecewise algebraic surfaces of low degree [10]. Sanglikar et al. employed differential geometry to formulate equations for blending parametric surfaces and gave closed form analytic solutions for most of the surfaces used in current solid modellers [11]. Cheng et al. proposed a finite difference method to solve fourth order partial differential equations and applied the solution in surface blending of quadric surfaces [12]. Ohkura and Kakazu described a generalized method for the blending of three primary surfaces which yields the formulae for both convex and concave combinations. They also presented the projective potential method to produce a convex combination of three surfaces [13]. Based on the determination of intersection of two offset surfaces using only the first-order derivatives of the progenitors, Barnhill et al. developed a method for blending two primary parametric surfaces [14]. Vida et al. presented a comprehensive review of blending between parametric surfaces, discussed the applicability and efficiency of parametric techniques for general blending situations, and pointed out open questions for future research in surface blending [15]. Defining the blending surface as a collection of intersection curves of correlated pairs of surfaces, Hartmann proposed a method to blend an implicit surface with a parametric surface which can achieve G^2-continuous transitions from the blending surface to primary surfaces by introducing a simple additional condition [16]. Chuang et al. used the derived spine curve and linkage curves to compute a parametric form of the variable-radius spherical and circular blends and investigated the variable-radius rolling-ball blending methods [17]. They translated geometric constraints specifying the variable radius into a non-linear system that represents the spine curve exactly for solving the difficulty in specifying the radius and tracing the spine curve [18]. Chunag and Lien also proposed an exact formulation which represents the blending in a high dimensional space as the swept surface of the intersection between the offsets of the base surfaces whose the radii satisfy a specific one-parameter curve, and another complex formulation defining the blend as the sweeping surface of the intersection between offsets of base surfaces whose radii satisfy a specific two-parameter surface [19]. Li and Li and Chang developed boundary penalty finite element methods to solve partial differential equations and applied it in surface blending [20-22]. On the basis of the theory of envelopes and discriminant sets, Lukács analyzed variable radius rolling ball (VRRB) blend surfaces which are envelopes of one parameter families of varying radius balls and usually G^1 continuous but often only piecewise curvature continuous [23]. Hartmann investigated a numerical implicitization for both parametric surfaces and many other surfaces of practical use which have no standard (parametric or implicit) representations in a uniform way as implicit surfaces [24]. Kós and colleagues examined the methods for recovering constant radius rolling-ball blends in reverse engineering [25]. Chen et al. gave a scheme to construct a piecewise algebraic surface which blends the given surfaces along the intersection curves of the given surfaces and their corresponding auxiliary planes [26]. You et al. discussed a fast resolution method for fourth order partial differential equations and applied it in blends of parametric surfaces [27]. Based on the geometric information of the given boundary curves, Chang and Yang used a boundary-blending method achieve unidirectional 2D parametrization and extended it to bi-directional parametrization via superposition to include both boundary pairs which leads to reasonable smooth blending of the boundaries [28]. Foufou and Garnier discussed how to determine principal circles tangent to both quadrics being blended, proposed a novel method to define G^1 Dupin cyclide blends between quadric primitives [29]. Han and Wu introduced the parametrization of implicit blending surfaces and investigated the subdivision scheme of the k^{th}-degree NURBS curve, and presented the algorithm of subdivision scheme for the implicit algebraic surfaces [30]. Song et al. developed a method to created n-sided (n = 3, 5, 6) G^2 blending surfaces with the exception of the 3-sided blending surface being C^0 continuous at the three vertexes and the 5-sided blending surface being C^0 continuous at a vertex [31].

In this paper, we will introduce the resolution of second order partial differential equations and examine how different orders of partial differential equations affect the smoothness at the transition curves between the blending surface and the primary surfaces represented in a parametric form.

2. Blending models with different orders of partial differential orders

Partial differential equations have been widely applied in surface blending. It is well known that a second order partial differential equation can only ensure the positional continuity. A fourth order partial differential equation can guarantee both positional and tangential continuities and a sixth order partial differential equation has a capacity to satisfy up to curvature continuities. However, the higher the order of the partial differential equation, the more difficult and inefficient it is to solve the equation. Mixed order partial differential equations can reduce the computational cost. A combination of two second and one fourth order partial differential equations have been applied in surface blending by Bloor and Wilson [5].

In the following, we will present a solution to a second order partial differential equation subject to blending boundary conditions, and combine the second and fourth order, second and sixth order, and fourth and sixth order partial differential equations as well as the second, fourth and sixth order partial differential equations to create blending surfaces of the same blending task. We will also examine the effects of these partial differential equations on blending surfaces.

Mathematically, surface blending with up to sixth order partial differential equation can be defined below.

For surface blending with second order partial differential equations, the resolution equations and blending boundary conditions are

$$\begin{aligned}
&\left(a_t \frac{\partial^2}{\partial u^2} + b_t \frac{\partial^2}{\partial v^2}\right) t(u,v) = 0 \\
&u = 0 \qquad t = t_0(v) \\
&u = 1 \qquad t = t_1(v) \\
&(t = x, y, z)
\end{aligned} \tag{1}$$

where u and v are parametric variables, a_t and b_t are shape parameters, x, y and z are position functions of blending surfaces, and $\mathrm{t}_0(v)$ and $\mathrm{t}_1(v)$ are boundary curves.

For surface blending with fourth order partial differential equations, the above mathematical model becomes

$$\begin{aligned}
&\left(a_t \frac{\partial^4}{\partial u^4} + b_t \frac{\partial^4}{\partial u^2 \partial v^2} + c_t \frac{\partial^4}{\partial v^4}\right) t(u,v) = 0 \\
&u = 0 \qquad t = t_0(v) \qquad \frac{\partial t}{\partial u} = t_1(v) \\
&u = 1 \qquad t = t_2(v) \qquad \frac{\partial t}{\partial u} = t_3(v) \\
&(t = x, y, z)
\end{aligned} \tag{2}$$

When curvature continuity is required, surface blending can be represented by

$$\begin{aligned}
&\left(a_t \frac{\partial^6}{\partial u^6} + b_t \frac{\partial^6}{\partial u^4 \partial v^2} + c_t \frac{\partial^6}{\partial u^2 \partial v^4} + d_t \frac{\partial^6}{\partial v^6}\right) t(u,v) = 0 \\
&u = 0 \qquad t = t_0(v) \qquad \frac{\partial t}{\partial u} = t_1(v) \qquad \frac{\partial^2 t}{\partial u^2} = t_2(v) \\
&u = 1 \qquad t = t_3(v) \qquad \frac{\partial t}{\partial u} = t_4(v) \qquad \frac{\partial^2 t}{\partial u^2} = t_5(v) \\
&(t = x,y,z)
\end{aligned} \tag{3}$$

By introduction of mixed order blends, three combinations of different orders of partial differential equations up to the sixth order exist. Usually, the x and y components are taken to be a lower order and the z component is set to a higher order. The first combination is from two second and one fourth order partial differential equations with the form of

$$\left(a_t \frac{\partial^2}{\partial u^2} + b_t \frac{\partial^2}{\partial v^2}\right) t(u,v) = 0$$
$$u = 0 \qquad t = t_0(v)$$
$$u = 1 \qquad t = t_1(v)$$
$$(t = x, y) \tag{4}$$
$$\left(a_z \frac{\partial^4}{\partial u^4} + b_z \frac{\partial^4}{\partial u^2 \partial v^2} + c_z \frac{\partial^4}{\partial v^4}\right) z(u,v) = 0$$
$$u = 0 \qquad z = z_0(v) \qquad \frac{\partial z}{\partial u} = z_1(v)$$
$$u = 1 \qquad z = z_2(v) \qquad \frac{\partial z}{\partial u} = z_3(v)$$

The combination of two second with one sixth order partial differential equations leads to the following blending mathematical model

$$\left(a_t \frac{\partial^2}{\partial u^2} + b_t \frac{\partial^2}{\partial v^2}\right) t(u,v) = 0$$
$$u = 0 \qquad t = t_0(v)$$
$$u = 1 \qquad t = t_1(v)$$
$$(t = x, y) \tag{5}$$
$$\left(a_z \frac{\partial^6}{\partial u^6} + b_z \frac{\partial^6}{\partial u^4 \partial v^2} + c_z \frac{\partial^6}{\partial u^2 \partial v^4} + d_z \frac{\partial^6}{\partial v^6}\right) z(u,v) = 0$$
$$u = 0 \qquad z = z_0(v) \qquad \frac{\partial z}{\partial u} = z_1(v) \qquad \frac{\partial^2 z}{\partial u^2} = z_2(v)$$
$$u = 1 \qquad z = z_3(v) \qquad \frac{\partial z}{\partial u} = z_4(v) \qquad \frac{\partial^2 z}{\partial u^2} = z_5(v)$$

Finally, the combination between two fourth and one sixth order partial differential equations result in surface blending representation as follows

$$\left(a_t \frac{\partial^4}{\partial u^4} + b_t \frac{\partial^4}{\partial u^2 \partial v^2} + c_t \frac{\partial^4}{\partial v^4}\right) t(u,v) = 0$$
$$u = 0 \qquad t = t_0(v) \qquad \frac{\partial t}{\partial u} = t_1(v)$$
$$u = 1 \qquad t = t_2(v) \qquad \frac{\partial t}{\partial u} = t_3(v)$$
$$(t = x, y) \tag{6}$$
$$\left(a_z \frac{\partial^6}{\partial u^6} + b_z \frac{\partial^6}{\partial u^4 \partial v^2} + c_z \frac{\partial^6}{\partial u^2 \partial v^4} + d_z \frac{\partial^6}{\partial v^6}\right) z(u,v) = 0$$
$$u = 0 \qquad z = z_0(v) \qquad \frac{\partial z}{\partial u} = z_1(v) \qquad \frac{\partial^2 z}{\partial u^2} = z_2(v)$$
$$u = 1 \qquad z = z_3(v) \qquad \frac{\partial z}{\partial u} = z_4(v) \qquad \frac{\partial^2 z}{\partial u^2} = z_5(v)$$

3. Resolution of PDE based blending problems

The blending problems described by Eq. (1) to (6) can be summarized into the separate resolution of second, fourth and sixth order partial differential equations subject to corresponding blending boundary conditions.

Similar to the treatment in [32, 33], the functions of boundary curves, tangents and curvatures can be divided into some independent basic functions. Taking the x component as an example, the decomposed boundary conditions for the positional continuity only become

$$\begin{aligned} u &= 0 \qquad x = \sum_{j=0}^{J_x} a_{x0j} g_{xj}(v) \\ u &= 1 \qquad x = \sum_{j=0}^{J_x} a_{x1j} g_{xj}(v) \end{aligned} \tag{7}$$

where a_{x0j} and a_{x1j} are known constants determined by the blending boundary conditions, and $g_{xj}(v)$ are independent basic functions.

Still taking the x component as an example, the position function of blending surfaces can be taken to be a combination of the functions of boundary curves with an unknown function of u parametric variable for the resolution of the second order partial differential equation

$$x(u,v) = \sum_{j=0}^{J_x} \sum_{m=0}^{M_{xj}} p_{jm} u^m g_j(v) \tag{8}$$

where p_{jm} are unknown constants.

Substituting Eq. (8) into (7) and solving for p_{j0} and p_{j1}, we obtain

$$\begin{aligned} p_{j0} &= a_{x0j} \\ p_{j1} &= a_{x1j} - a_{x0j} - \sum_{m=2}^{M_{xj}} p_{jm} \\ (j &= 0,1,2,\cdots,J_x) \end{aligned} \tag{9}$$

Introducing Eq. (9) back to the position function (8), boundary conditions (7) are satisfied exactly and (8) becomes

$$x(u,v) = \sum_{j=0}^{J_x} \left\{ a_{x0j} + (a_{x1j} - a_{x0j})u + \sum_{m=2}^{M_{xj}} p_{jm}(u^{m-1} - 1)u \right\} g_j(v) \tag{10}$$

The error function of Eq. (1) is reached by considering Eq. (10) which has the form of

$$\begin{aligned} R(u,v) = \sum_{j=0}^{J_x} \Bigg\{ & \sum_{m=2}^{M_{xj}} p_{jm} \left[m(m-1) a_x u^{m-2} g_j(v) \right. \\ & \left. + b_x (u^m - u) g_j''(v) \right] + b_x \left[a_{x0j} + (a_{x1j} - a_{x0j})u \right] g_j''(v) \Bigg\} \end{aligned} \tag{11}$$

In order to minimize the error of partial differential equation (1), some collocation points are chosen and the square sum of the error function at these collocation points is calculated. Its differentiation with respect to the unknown constants p_{jm} $(m = 2,3,4,\cdots M_{xj})$ produces the following linear algebraic equations

$$\begin{aligned} & \sum_{j=0}^{J_x} \left(\sum_{m=2}^{M_{xj}} K_{ljm} p_{jm} + C_{lj} \right) = 0 \\ & \left[l = 1,2,3\cdots,(J_x+1)(M_{xj}-1) \right] \end{aligned} \tag{12}$$

For the fourth and sixth order partial differential equations, the position function is still taken to be (8). With the similar treatment, the resultant linear algebraic equations can be obtained and have the same form as (12).

Solving Eq. (12), the x component is determined. Carrying out the same operation, both y and z components can be determined. Then, these position functions are used to generate blending surfaces.

4. Effects on blending surfaces

In this section, we will use some examples to study how different orders of partial differential equations affect the continuities of blending surfaces. The first example is to blend the frustum of an irregular cone with an elliptic cylinder. The boundary conditions up to curvature continuities for this blending problem can be written as

$$
\begin{aligned}
u=0 \quad & x = Ru_0 \cos v \\
& \frac{\partial x}{\partial u} = R' \cos v \qquad \frac{\partial^2 x}{\partial u^2} = 0 \\
& y = Ru_0 v \sin v \\
& \frac{\partial y}{\partial u} = R' v \sin v \qquad \frac{\partial^2 y}{\partial u^2} = 0 \\
& z = h_0 u_0 \\
& \frac{\partial z}{\partial u} = h_0' \qquad \frac{\partial^2 z}{\partial u^2} = 0 \\
u=1 \quad & x = x_0 + a\cos\alpha\cos v + h_1 u_{01} \sin\alpha \\
& \frac{\partial x}{\partial u} = h_1' \sin\alpha \qquad \frac{\partial^2 x}{\partial u^2} = 0 \\
& y = b \sin v \\
& \frac{\partial y}{\partial u} = 0 \qquad \frac{\partial^2 y}{\partial u^2} = 0 \\
& z = z_0 - a\sin\alpha\cos v + h_1 u_{01} \cos\alpha \\
& \frac{\partial z}{\partial u} = h_1' \cos\alpha \qquad \frac{\partial^2 z}{\partial u^2} = 0
\end{aligned}
\tag{13}
$$

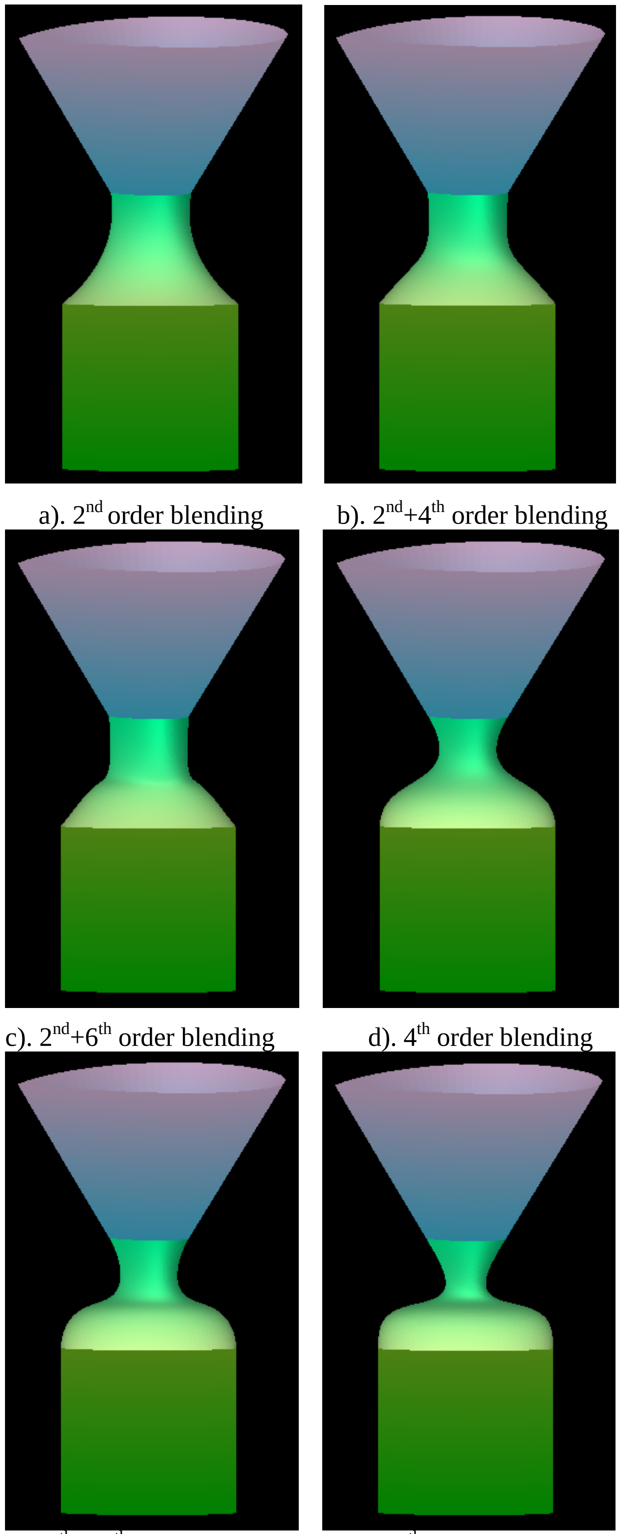
With the above methods, the blending surface from Eq. (1) to (6) is given in Figure 1, respectively.

It can be seen from these figures that the second order partial differential equations can only create blending surface with positional continuity. Keeping partial differential equation for x and y components to be the second order and changing the partial differential equation for the *z* component to a fourth order do not improve the continuity between the blending surface and primary surfaces. Even worse, a higher order partial differential equation for the *z* component appears to bring in poorer results at the boundary curves. This can be observed from the blending surface in Figure 1c which is produced by the combination between 2 second and one sixth order partial differential equations.

However, when all the partial differential equations become the fourth order, the smooth transition from the blending surface to primary surfaces is created. This is because both positional and tangential continuities at boundary curves are achieved.

Further changing all the partial differential equations to the sixth order, the blending surface in Figure 1e was obtained. It is smoother than that in Figure 1d because of the introduction of curvature continuity.

For the mixed order blends, when the lowest order of partial differential equations is larger than 4, higher order partial differential equation for the z component leads to better continuity. This argument is supported by the observation of Figures 1d and 1e.

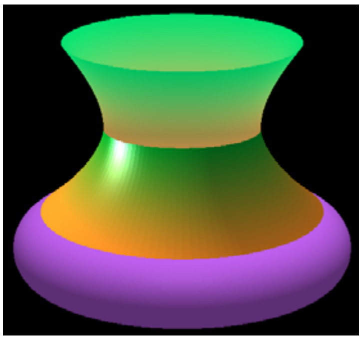


a). 2nd order blending; b). 2nd+4th order blending; c). 2nd+6th order blending; d). 4th order blending; e). 4th + 6th order blending; f). 6th order blending

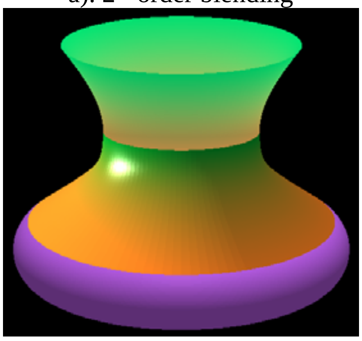
Figure 1 Blending between the frustum of an irregular cone and an elliptic cylinder

The second example is to blend a circular torus and an elliptic hyperboloid of one sheet. The boundary conditions including curvature continuity have the form of

$$
\begin{aligned}
u=0 \quad & x = a\cosh u_0 \cos v \\
& \frac{\partial x}{\partial u} = a\sinh u_0 \cos v \qquad \frac{\partial^2 x}{\partial u^2} = a\cosh u_0 \cos v \\
& y = b\cosh u_0 \sin v \\
& \frac{\partial y}{\partial u} = b\sinh u_0 \sin v \qquad \frac{\partial^2 y}{\partial u^2} = b\cosh u_0 \sin v \\
& z = h_0 + h\sinh u_0 \\
& \frac{\partial z}{\partial u} = h\cosh u_0 \qquad \frac{\partial^2 z}{\partial u^2} = h\sinh u_0 \\
u=1 \quad & x = (R + A\cos u_1)\cos v \\
& \frac{\partial x}{\partial u} = -A\sin u_1 \cos v \qquad \frac{\partial^2 x}{\partial u^2} = -A\cos u_1 \cos v \\
& y = (R + A\cos u_1)\sin v \\
& \frac{\partial y}{\partial u} = -A\sin u_1 \sin v \qquad \frac{\partial^2 y}{\partial u^2} = -A\cos u_1 \sin v \\
& z = A\sin u_1 \\
& \frac{\partial z}{\partial u} = A\cos u_1 \qquad \frac{\partial^2 z}{\partial u^2} = -A\sin u_1
\end{aligned}
\tag{14}
$$



a). 2nd order blending



b). 6th order blending

Figure 2 Blending between a circular torus and an elliptic hyperboloid of one sheet

In the interest of space, here we only consider the blending surfaces from the second and sixth order partial differential equations, respectively. Using the above methods, the blending surface from the second order partial differential equations was shown in Fig. 2a and that by the sixth order partial differential equations was depicted in Figure 2b.

Once again, the images in Figures 2a and 2b indicate that the sixth order partial differential equations generate smoother transition between the blending surface and primary surfaces than the second order partial differential equations. However, although both blends in Figure 1a and Figure 2a employed the second order partial differential equations, the continuity in Figure 2a is better than that in Figure 1a. This suggests that for some blending tasks, i.e., when the tangential information of the blending surfaces defined by Eq. (1) at the boundary curves is close to that of primary surfaces, the second order partial differential equations can create visible smoothness between the blending surface and primary surfaces.

5. Conclusions

Different orders of partial differential equations determine the continuity between the blending surface and primary surfaces. In this paper, we investigate this issue.

We discussed the mathematical models of the surface blending with different orders and mixed orders of partial differential equations and the resolution of the second order partial differential equations subject to blending boundary conditions.

The order of partial differential equations has a strong influence on the continuity of blending surfaces. For general cases, the second order partial differential equations cannot achieve satisfactory smoothness between the blending surface and primary surfaces. For such a situation, the smoothness cannot be improved by mixed order blends. In contrast, when the order of partial differential equations is not less than 4, the introduction of higher order partial differential equation for one position component is helpful for a smoother transition from the blending surface to primary surfaces.

Acknowledgements

This research is funded by the British Arts and Humanities Research Council grant B/RG/AN5263 /APN12727.

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