

# Illumination Models

# Illumination Models

*Illumination models* are used to generate the colour of an object's surface at a given point on that surface.

The factors that govern the *illumination model* determine the visual representation of that surface.

Due to the relationship defined in the model between the surface of the objects and the lights affecting it, *illumination models* are also called *shading models* or *lighting models*.

# Light

The physics of light is a complex subject.

Shading models are approximations of these laws, in varying levels of realism/complexity.

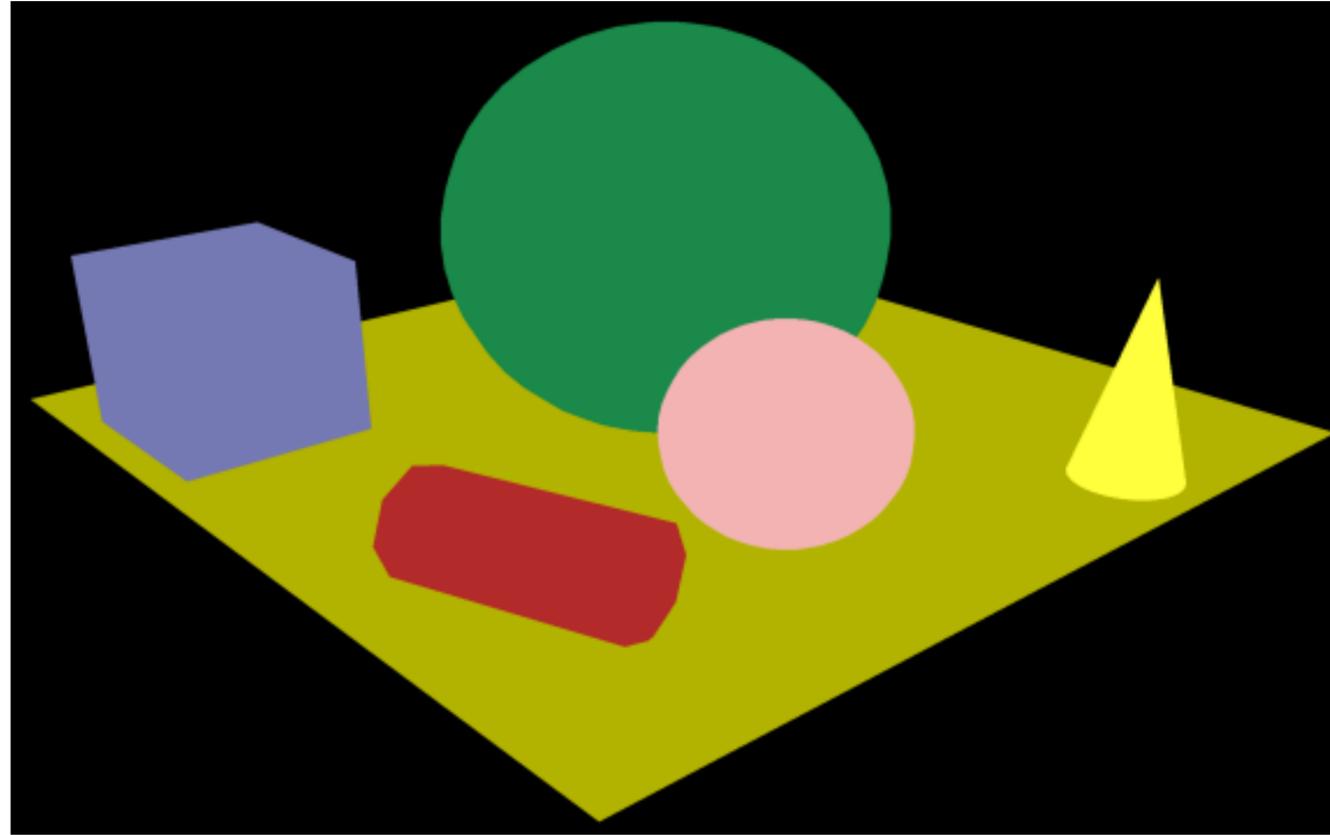
This is based on the fact that **surfaces**, for the most part, are approximations.

Micro facet details defines the lighting characteristics of surface colour.

CG Object representation (usually) does not transcend to that level.

Radiosity algorithms now mimic photonic reactions with surfaces.

# The Simplest Shading Model



The simplest shading model is a simple constant illumination.

The model represents an un-realistic assumption in that the surface is self illuminating (the colour of this constant shading is defined by the user).

# Illumination Equation

We can now introduce the *illumination equation*, using the notion that

$$I = \textit{value}$$

Where *I* represents the *illumination colour intensity*  
and

*value* represents the expression resulting in that colour value.

Constant shading illumination equation can be defined as:

$$I = k_i$$

...where *k* represents the basic object intensity.

# Constant Illumination

This simple equation has no reference to light sources, and as such every point on the object has the same intensity.

This equation need only be calculated once per object.

The process of evaluating the illumination equation at one or more points on an object's surface is referred to as *lighting* the object.

# Ambient Light

- Ambient light is usually a scene-based intensity of non directional light.
- It affects every object in that scene.
- We can then incorporate this into our illumination equation...

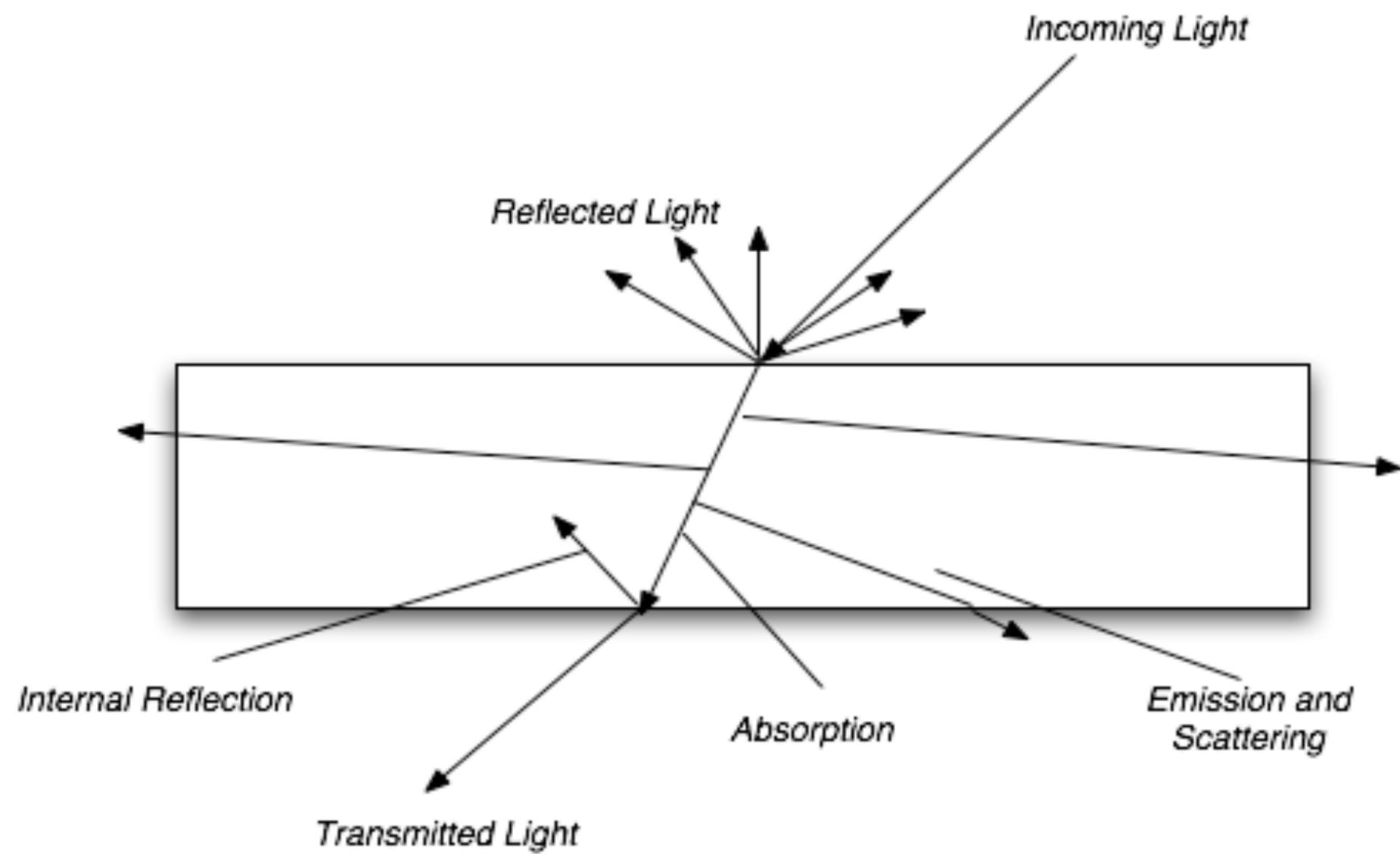
$$I = I_a k_a$$

- $I_a$  is the ambient light intensity the scene-based value that remains the same for all surfaces in the scene.
- $k_a$  is the ambient reflection coefficient a material based property that determines how much ambient light is actually reflected.
- Material based properties are the properties that characterise one surface from another.
- This equation allows individual surface properties to “reflect” a level of ambient light.

# BRDF

- Bidirectional Reflectance Distribution Function
- It describes how much light is reflected when light makes contact with a material (and hence can be used to specify different material types)
- The degree to which light is reflected depends on the viewer and light position relative to the surface normal and tangent
- BRDF is a function of incoming (light) direction and outgoing (view) direction relative to a local orientation at the light interaction point

# BRDF



# BRDF

- Depending upon the Wavelength of the Light there are different levels of absorption, reflection and transmission.
- Therefore BRDF is also dependent upon Wavelength.
- We need to specify the conservation of energy thus
- $\text{light incident at surface} = \text{Reflected Light} + \text{Absorbed Light} + \text{Transmitted Light}$

# BRDF

- Position of the light is also important as different areas of the surface will behave differently.
- This can be used to create surface detail, such as the rings and knots in wood.



# BRDF

- BRDF is usually specified as a function thus

$$BRDF_{\lambda}(\theta_i, \phi_i, \theta_o, \phi_o, u, v)$$

- $\lambda$  is the wavelength of the light
- $\theta_i, \phi_i$  is the incoming light in Spherical Co-ordinates
- $\theta_o, \phi_o$  is the outgoing reflected light in Spherical Co-ordinates
- $u, v$  is the position of the surface in parametrised in texture co-ordinate space

# BRDF

- If there is no positional invariance then the BRDF may be specified thus

$$BRDF_{\lambda}(\theta_i, \phi_i, \theta_o, \phi_o)$$

- This only works for materials with no surface variation (Homogeneous) and can be speeded up using lookup tables and texture modulation

# BRDF Definition

$$BRDF = \frac{L_o}{E_i}$$

- $w_i$  = incoming light direction
- $w_o$  = reflected light direction (outgoing)
- $L_o$  = quantity of reflected light in direction  $w_o$
- $E_i$  = quantity of light arriving from direction  $w_i$

# Types of BRDF

- There are two main types of BRDF
- isotropic BRDFs and anisotropic BRDFs
- The important properties of BRDFs are reciprocity and conservation of energy
- BRDFs that have these properties are considered to be physically plausible

# Isotropic BRDF

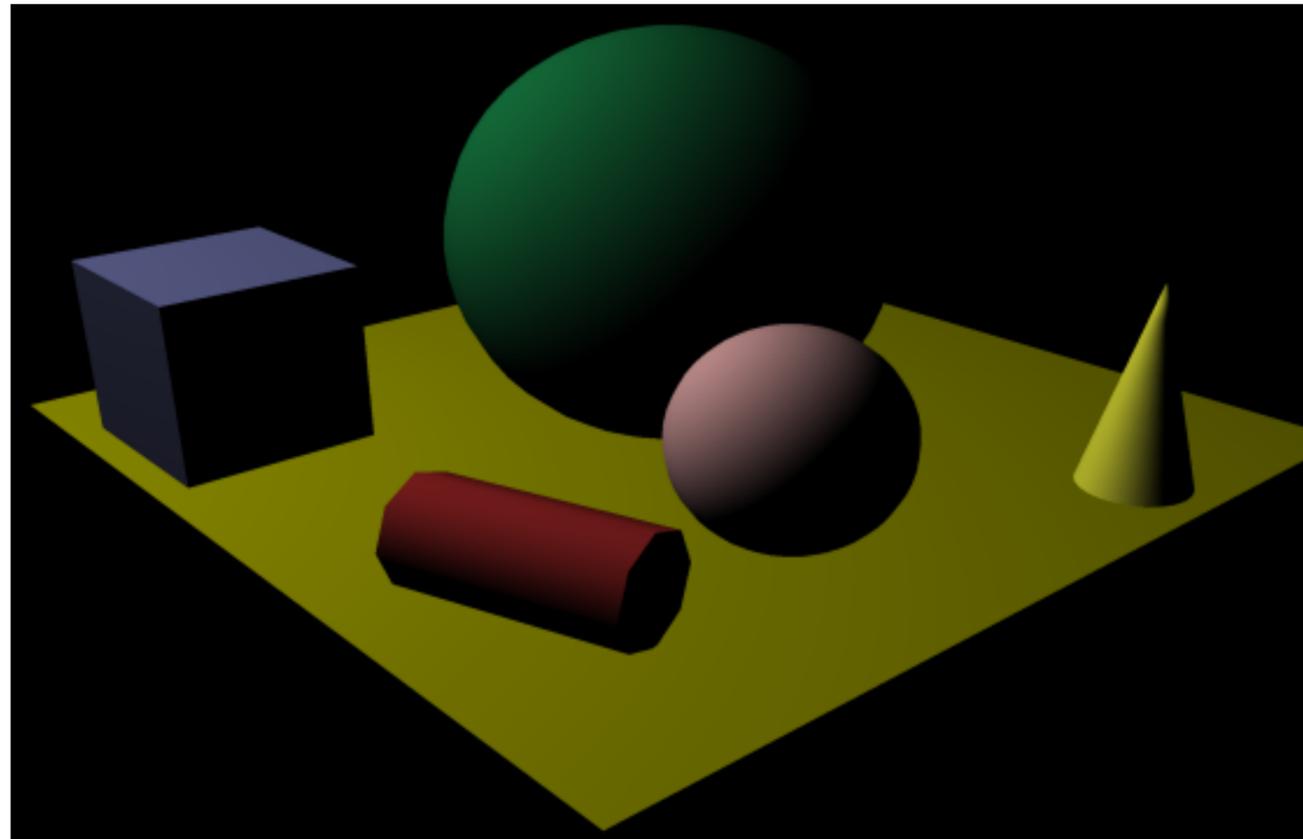
- These type of surfaces are invariant to with respect to rotation around the surface normal
- effectively this means that it should respond the same way from wherever we view it.
- and are quicker to calculate from the point of view of the renderer

# Anisotropic BRDF

- BRDFs that describes reflectance properties that do exhibit change with respect to rotation of the surface around the surface normal vector
- Anisotropy (the opposite of isotropy) is the property of being directionally dependent.
- Something which is anisotropic may appear different or have different characteristics in different directions.
- Seen in a materials such as Velvet

# Diffuse Reflection

Diffuse reflections consider point lights to generate shading properties  
a change in colour intensity across the surface of an object in relation to light sources.



The simplest of these models is the *Lambert Illumination Model*.

# Lambert's Law

*Lambertian Reflection* – light is reflected with equal intensity in all directions (isotropic).

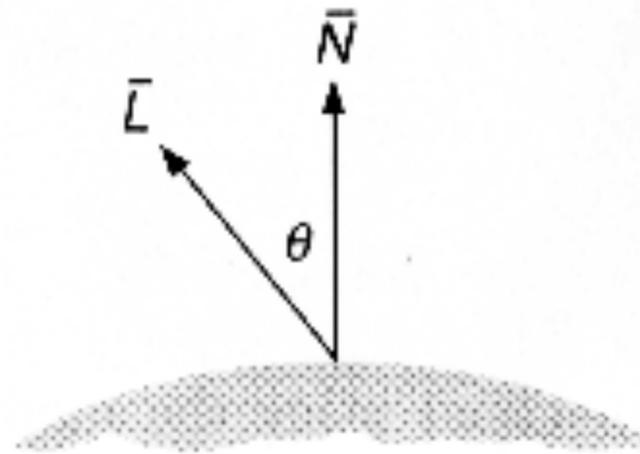
The distribution is a basic consideration towards surface detail:

Light scattering on the surface and in the medium.

# Lambert's Law

## Lambert's law states

*that the intensity of illumination on a diffuse surface is proportional to the cosine of the angle generated between the surface normal vector and the surface to the light source vector.*



The only data used in this equation is the surface normal and a light vector that uses the light source position (taken as a point light for simplicity).

The intensity is irrespective of the actual viewpoint, hence the illumination is the same when *viewed* from any direction.

# Lambert's Model

The equation for Lambert's illumination model is:

$$I = I_p k_d \cos(\theta)$$

Where:

$I_p$  is the *intensity* of the point light source

$k_d$  is the material *diffuse reflection coefficient* the amount of diffuse light reflected.

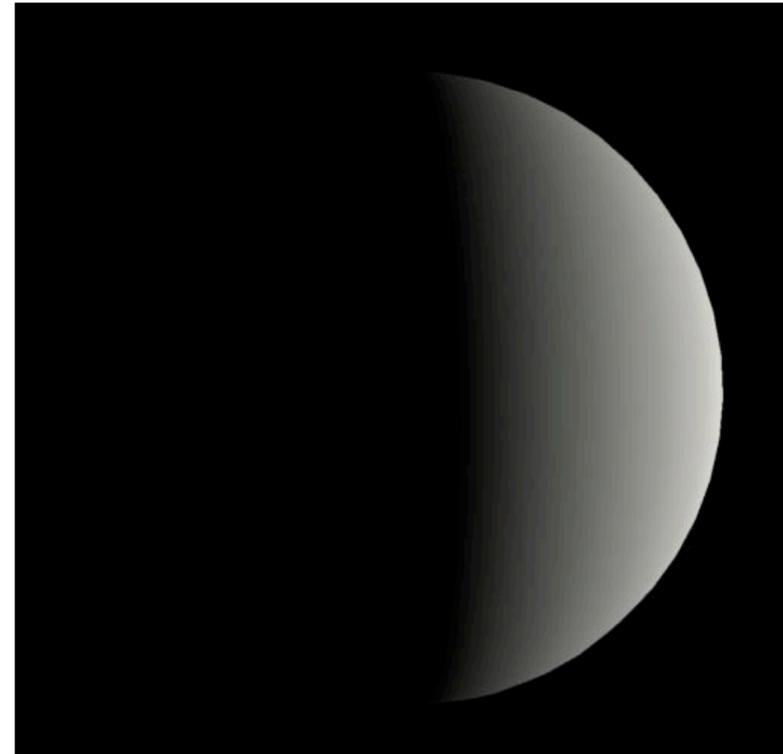
By using the dot product between 2 vectors  $v_1$  and  $v_2$

$$v_1 \bullet v_2 = |v_1| |v_2| \cos(\theta)$$

...and if  $N$  and  $L$  are normalised, we can re-write the illumination equation:

$$I = I_p k_d (N \bullet L)$$

# Problem with Lambert



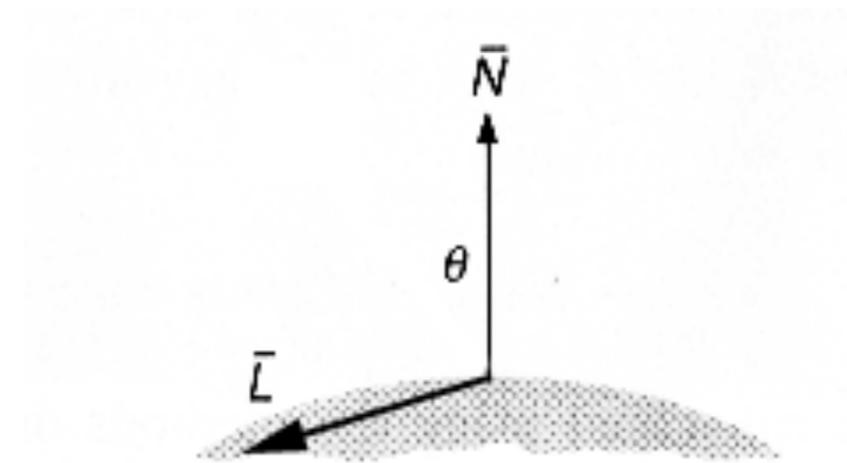
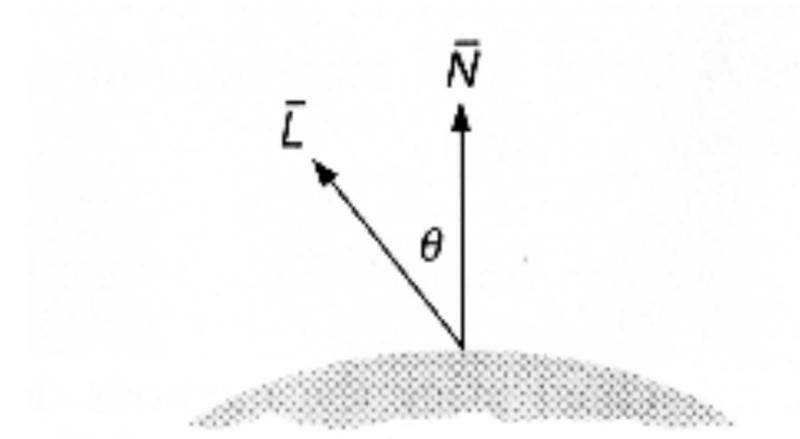
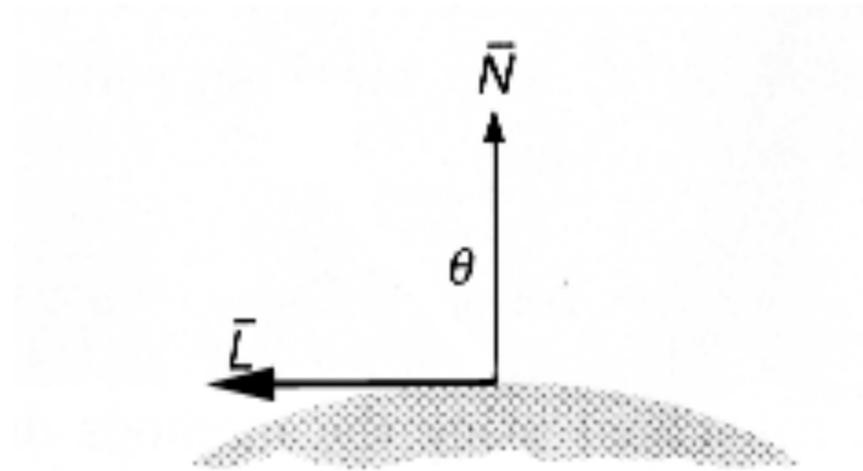
- You can see how Lambert's law is too general...

# Occlusion...

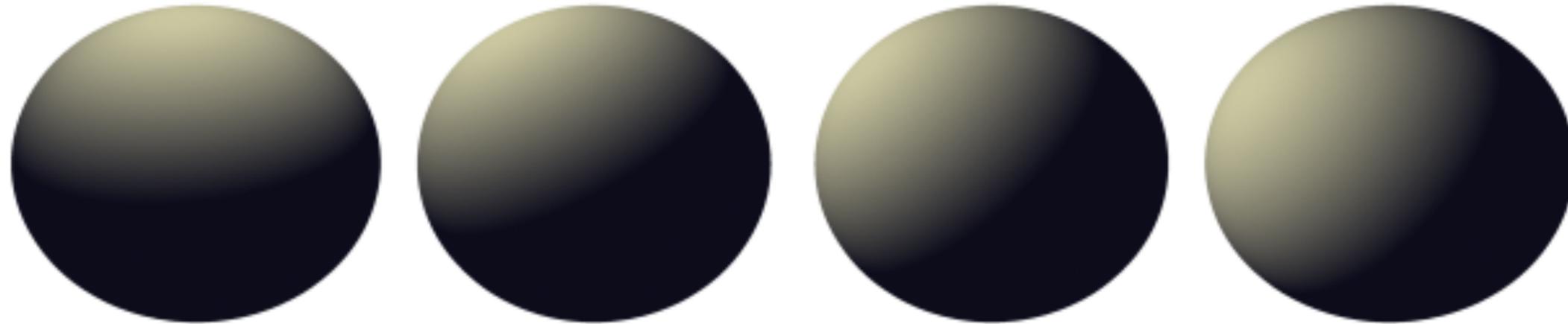
$$I = I_p k_d (N \cdot L)$$

Usually implies:

$$I = I_p k_d \max(N \cdot L, 0)$$



# Directional Light



So far, we have considered only *point lights*

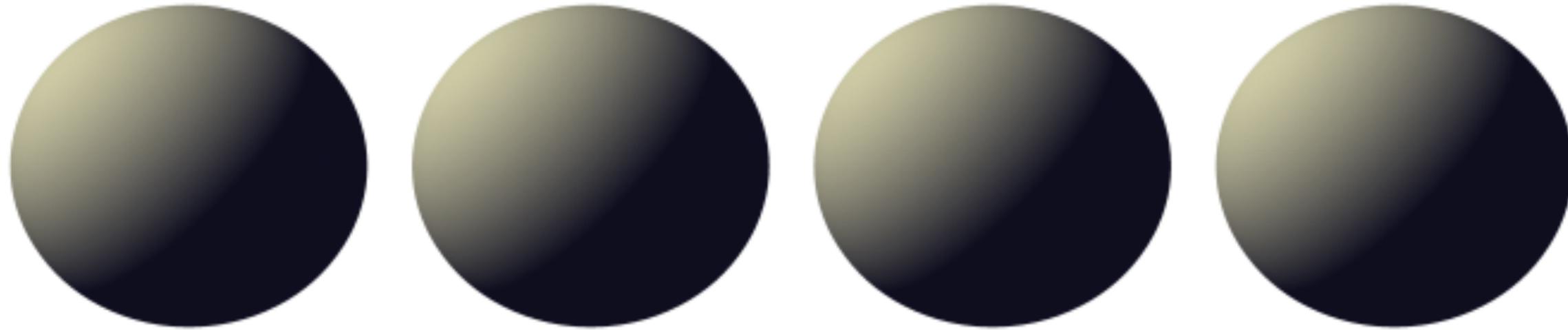
A light source at a near-infinite distance away from a surface has **near-parallel rays**.

The vector made from the surface to the light source is always the same for every surface.

This is known as a *directional light* – light from a known *direction*- not position.

We do not specify a position for directional lights, just a vector to indicate ray direction.

# Application of Directional Lights



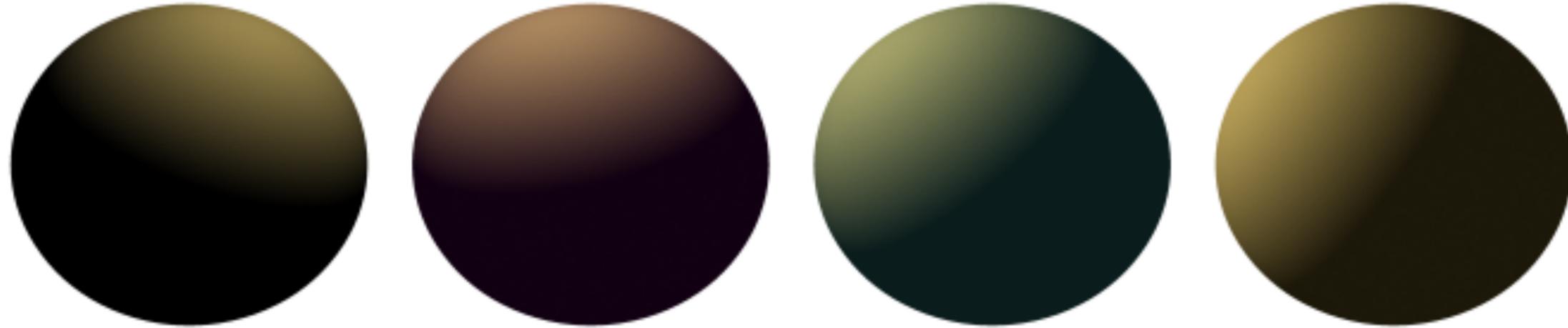
To implement a directional light into the illumination equation,

we simply use the same light vector in every different surface illumination equation –

i.e.  $\mathbf{L}$  does not change, as it is constant for the light source.

$$I = I_p k_d (\mathbf{N} \cdot \mathbf{L})$$

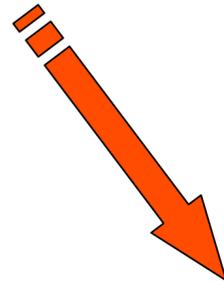
# Concatenation of Illuminations



*Lambert's lighting model* can be combined with the previous *ambient light equation*.

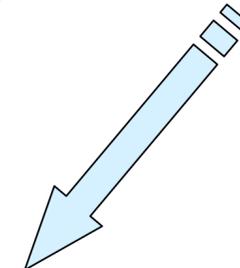
Ambient Light

$$I = I_a k_a$$



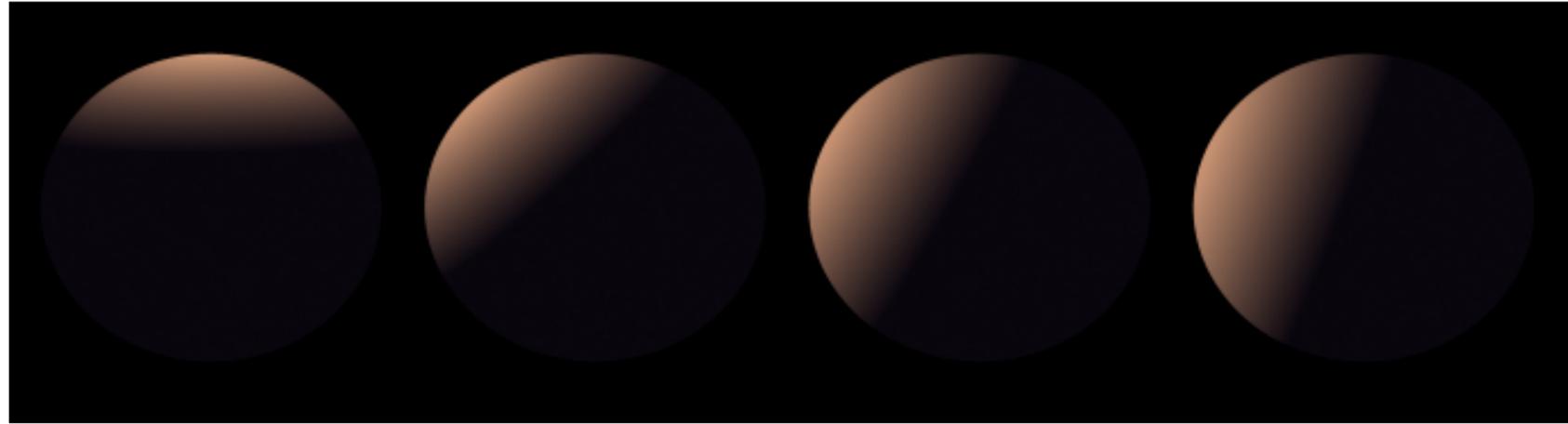
Lambert's

$$I = I_p k_d (N \cdot L)$$

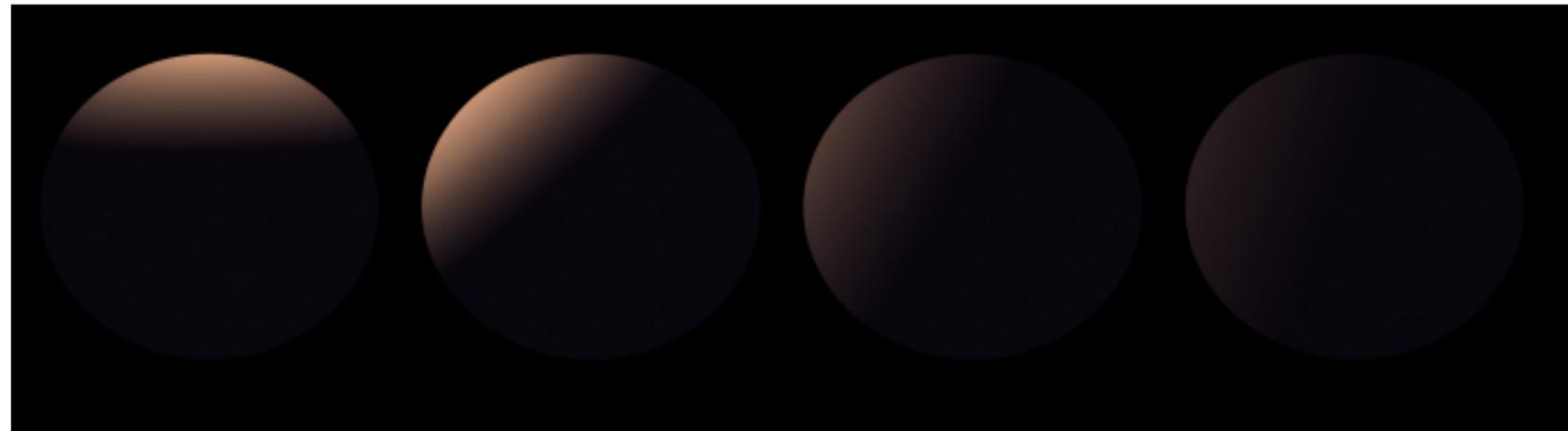


$$I = I_a k_a + I_p k_d (N \cdot L)$$

# Attenuation of Light



The above is an example of lighting with no attenuation and below is an example with a more realistic attenuation of light.



Real light has an *attenuation factor* – light intensity becomes weaker over distance.

In a similar manner, the perception of sound volume decreases the further away you are from its source.

# Application of Attenuation

To include attenuation into our illumination equation, we need to insert an *attenuation factor* into the lighting section – in this case labelled  $f_{att}$ .

$$I = I_a k_a + f_{att} I_p k_d (N \cdot L)$$

With this multiplying factor in place, we can control the intensity of light based on distance.

An  $f_{att}$  of 0 would effectively turn the light off.

An  $f_{att}$  of 1 would result in a maximum intensity of the light.

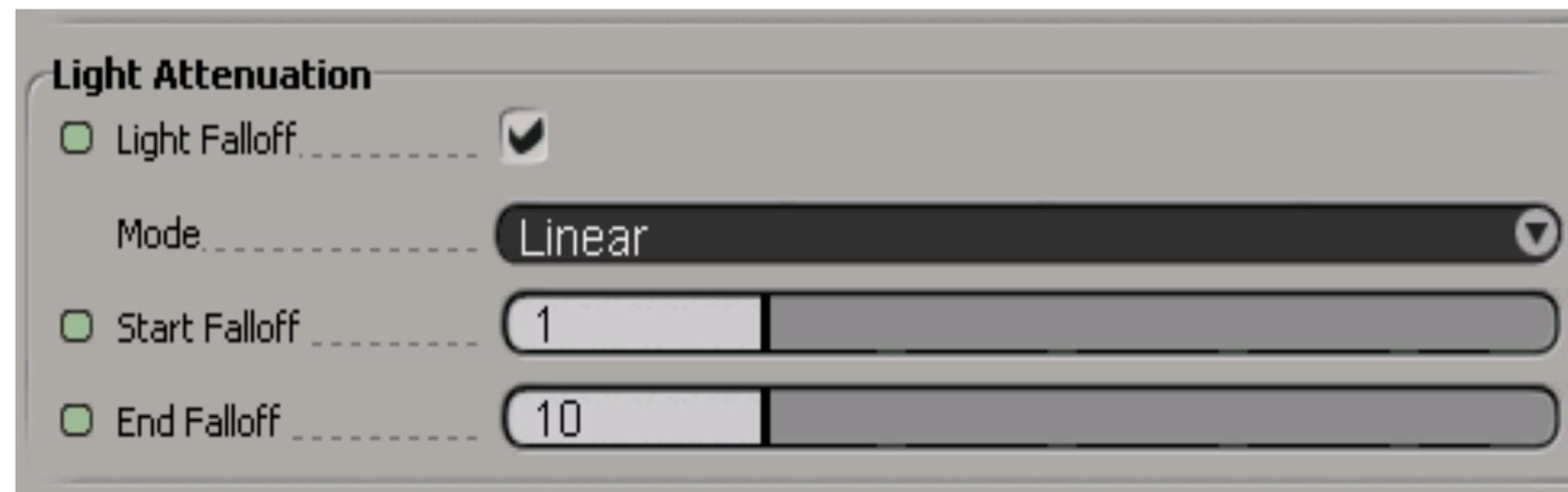
Light falloff obeys what is commonly known as the inverse square law – its intensity decreases exponentially in relation to distance.

If we consider the distance from the surface point to the light  $L$  as  $d_L$ ...

$$f_{att} = \frac{1}{d_L^2}$$

# XSI Attenuation

- This gives a small range of useable light and most CG applications use less realistic methods to light scenes.
- e.g. you can set the falloff as a linear amount, with the distances from which to range the falloff being defined by the user or even no fall off at all.



# Light Colour

So far the notion of using the illumination equation has had no reference to actual colour – only monochromatic intensity.

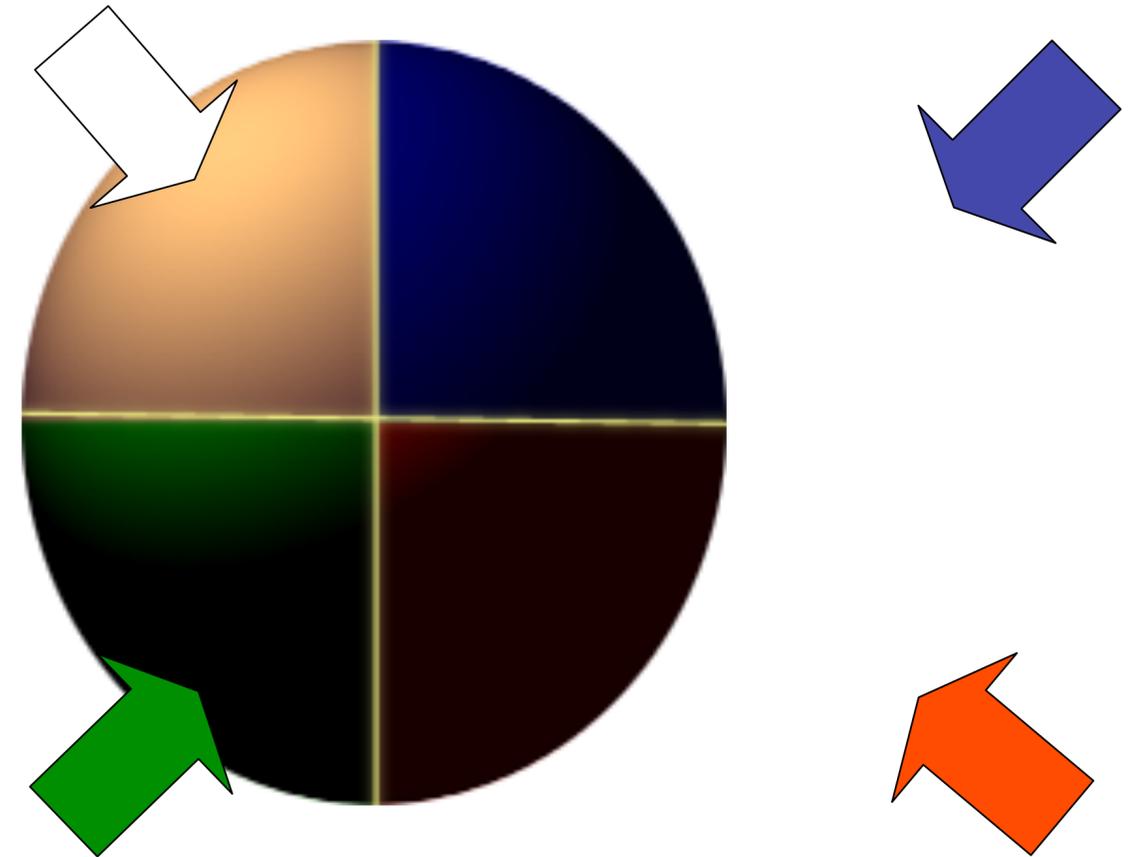
What we require to do so, are 3 equations, to represent the Red, Green and Blue data.

E.g.

$$I_R = I_{aR}k_{aR} + f_{att}I_{pR}k_{dR}(N \cdot L)$$

$$I_G = I_{aG}k_{aG} + f_{att}I_{pG}k_{dG}(N \cdot L)$$

$$I_B = I_{aB}k_{aB} + f_{att}I_{pB}k_{dB}(N \cdot L)$$



# Specular Reflection

Shiny surfaces exhibit *specular reflection* – the reflection of the light source towards the viewer.

Specular reflection has 2 main colour biases:

1. The *colour* of the specular reflection is determined by the *light source colour*.

or

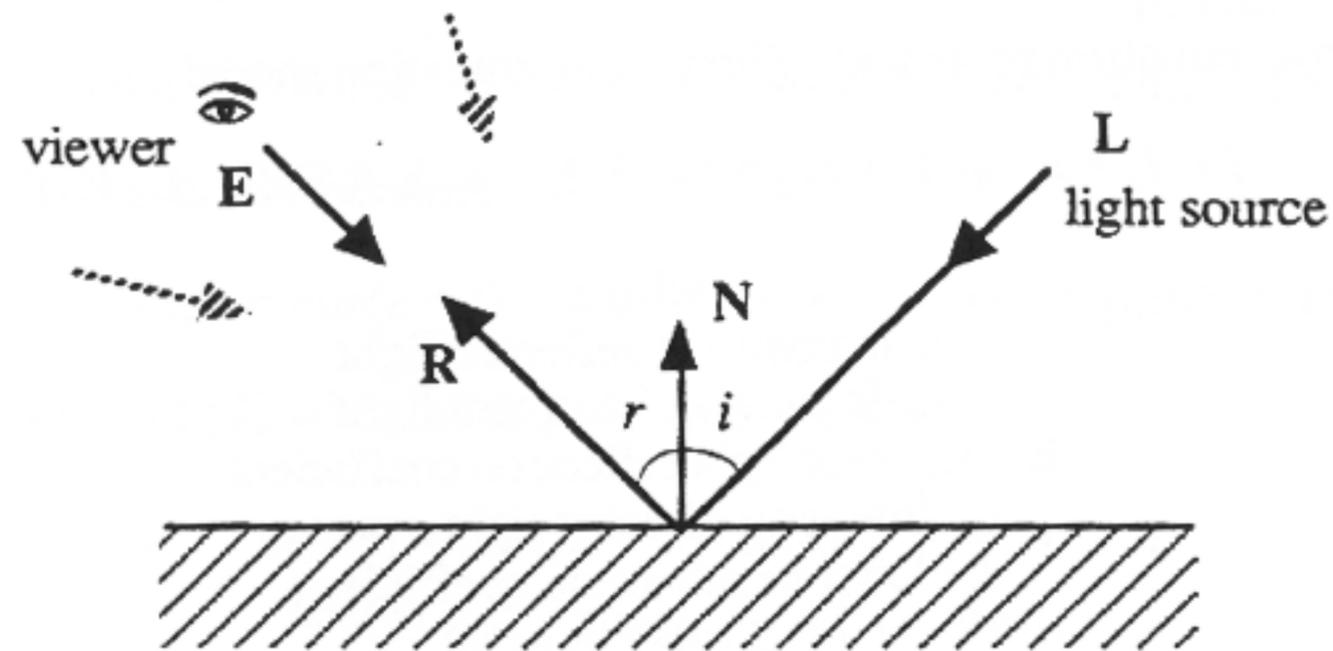
2. The *colour* of the specular reflection is determined by the *colour of the surface*.

Objects that have waxed, or transparent surfaces (apples, plastic, etc.) tend to reflect the colour of the light source.

Plastic, for example, is composed of colour pigments suspended in a transparent material.

...and Gold has a *gold* coloured highlight.

# Specular Realism



The above diagram describes a perfectly mirrored surface.

The viewer is seen looking at a point on the surface: viewing vector  $E$ .

The light source  $L$  emanates a ray of light that that hits the surface with an angle of  $i$  relative to the normal.

The angle of reflection of the light ray then leaves with an angle  $r$  relative to the normal.

The angle of incidence  $i$  is the same as angle  $r$ .

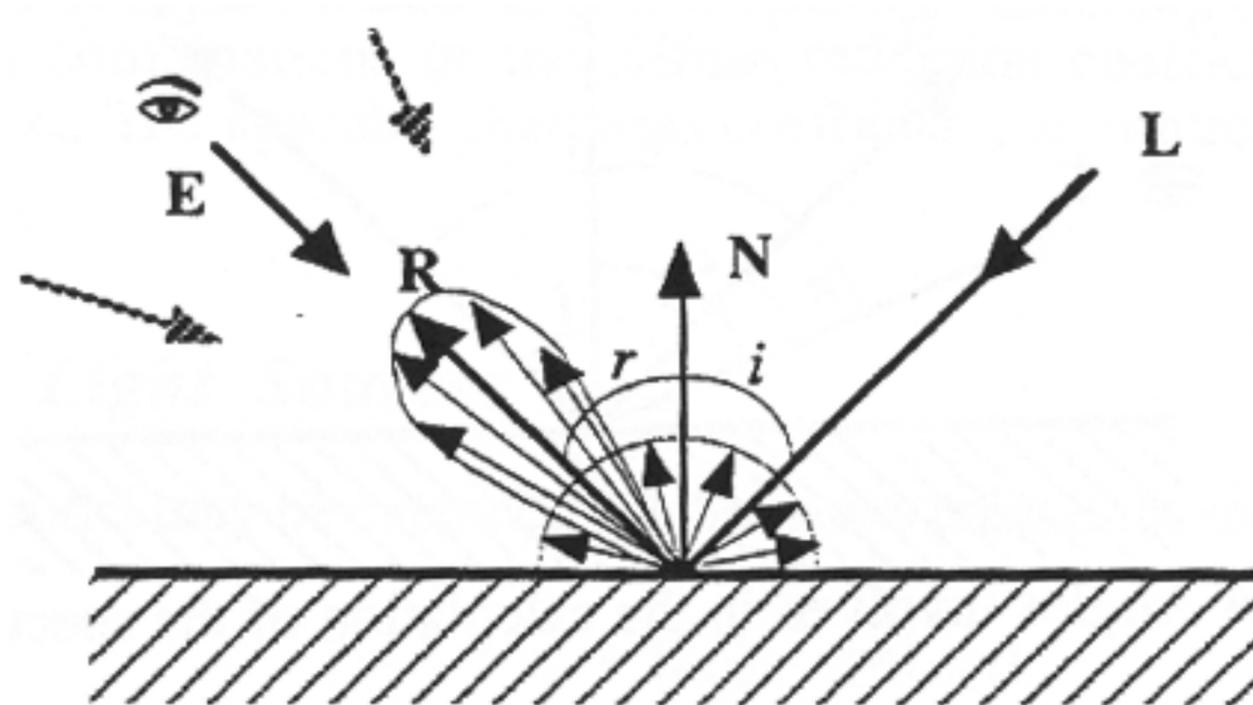
In a perfect mirror (above), the viewer may **ONLY** see the light ray if the viewing angle  $E$  is directly opposite of the reflection vector  $R$ .

# Specular Realism II

Most shiny surfaces are *not* perfect mirrors.

Shiny surfaces will reflect the largest intensity where the viewing angle is directly opposite the reflection angle.

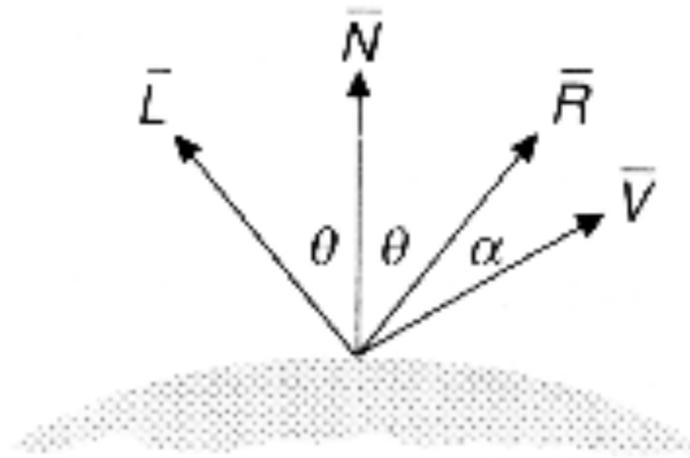
They usually also reflect a diminishing gradient of highlight as well, enabling light to be seen from angles not directly opposed to the angle of reflection.



**Phong Bui-Tuong** developed an illumination model for *non-perfect* reflectors that has become widely used to portray realistic shiny surfaces.

It is commonly known as the *Phong illumination model*.

# Phong's Model



In the above diagram, the angle of incidence in relation to the surface normal is theta.

$\vec{V}$  represents the viewing vector (reversed so we are looking from the surface)

*alpha* the angle between the viewing vector and the reflection vector.

Phong postulated that maximum specular reflection was achieved when the angle between the viewing angle and the the reflection angle was smallest – i.e. *alpha* is zero.

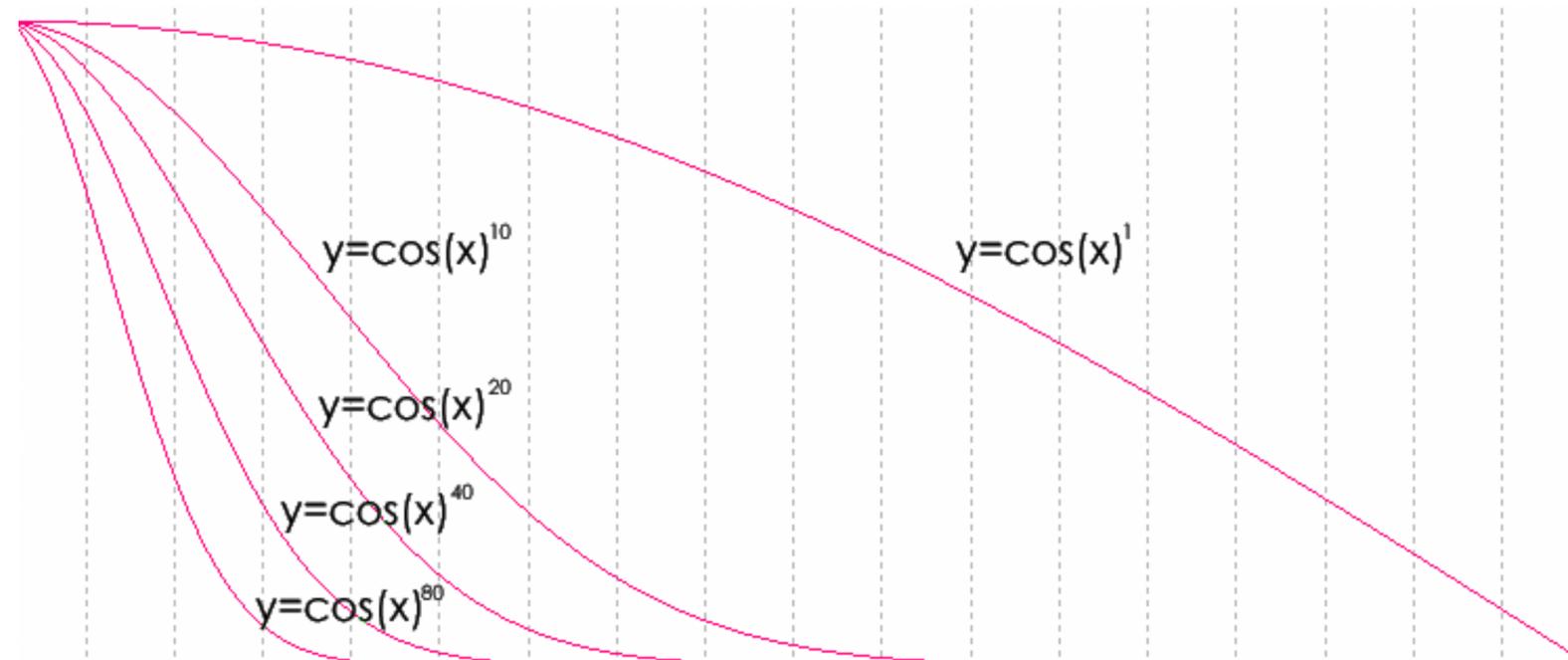
He then stipulated that the specular reflection falls off sharply as *alpha* increases –

which he stated could be represented by  $\cos^n \alpha$ .

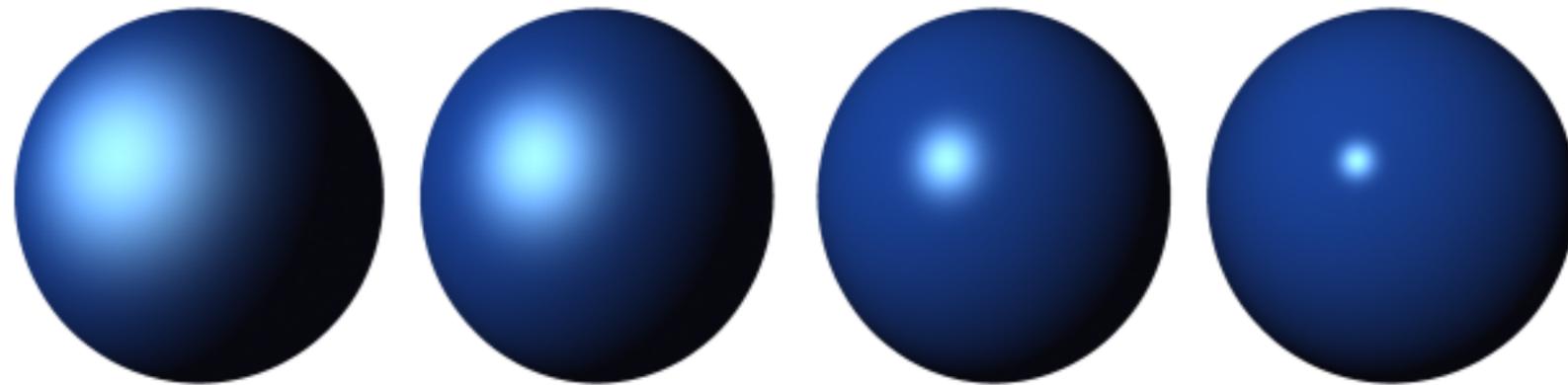
# Phong's Falloff

What does the *cos* <sup>n</sup> alpha mean?

Lets examine some graphical representations of various exponents of cos alpha.



# Phong's Equation



The above examples have Phong falloff values of 8, 16, 64 and 256 (from left to right).

The Phong Illumination Equation reads as follows:

$$I = \overset{\text{Ambient}}{I_a k_a} + \overset{\text{Lambert}}{f_{att} I_p k_d (N \cdot L)} + \overset{\text{Phong}}{f_{att} I_p k_s (R \cdot V)^n}$$

$k_s$  represent the specular reflection coefficient,  
 $n$  the exponent of the cosine function,

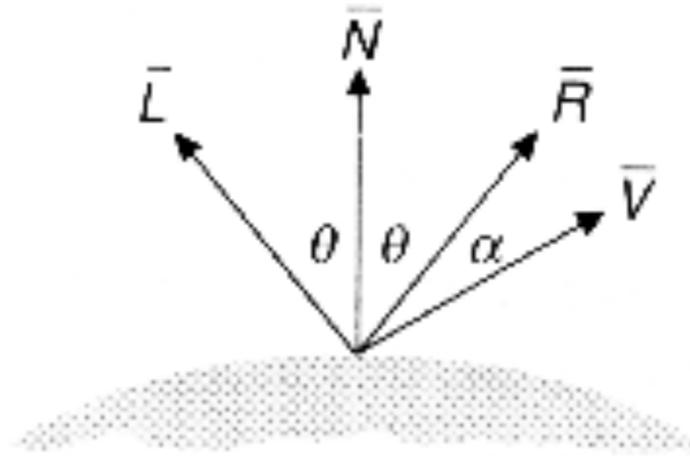
and the cosine of the angle between the *viewing vector* and the *reflection vector* can be calculated via the dot product of the 2 *normalised* respective vectors.

# Adding Phong to Illumination Model

Phong's contribution was in providing the specular aspect to the lighting model  
– we still maintain the Lambert and Ambient equations.

$$I = \overset{\text{Ambient}}{I_a k_a} + \overset{\text{Lambert}}{f_{att} I_p k_d (N \cdot L)} + \overset{\text{Phong}}{f_{att} I_p k_s (R \cdot V)^n}$$

# Geometry of Illuminations



$$I = \boxed{I_a k_a} + \boxed{f_{att} I_p k_d (N \cdot L)} + \boxed{f_{att} I_p k_s (R \cdot V)^n}$$

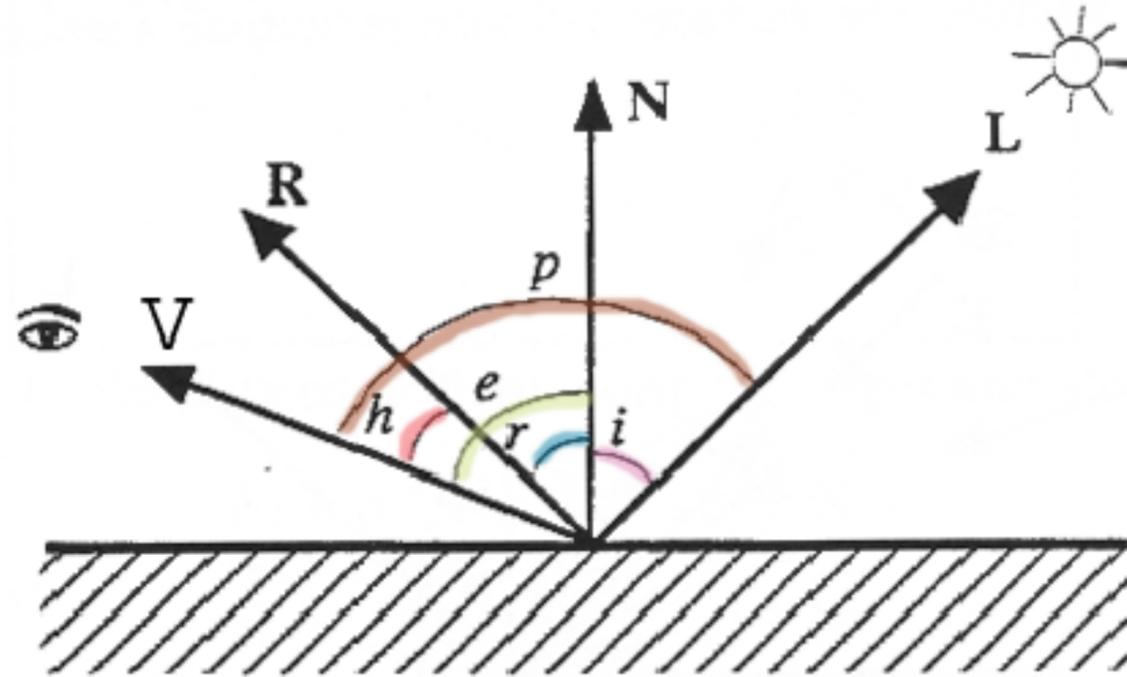
The equation is annotated with labels above the terms: "Ambient" in blue above the first term, "Lambert" in blue above the second term, and "Phong" in yellow above the third term.

The vectors used in Lambert's models are easily generated from information available – the surface normal, the surface point and the light position.

The vectors used in Phong's addition are less easily generated.

$V$  is fairly straightforward (the viewer position and surface point position are available).

The vector  $R$  however has to be generated from existing data.



The above diagram incorporates extra detail: ***h*** represents the angle between ***V*** and ***R***.

$$h = e - r = e - i$$

$$p = e + i$$

Thus using trigonometric functions:

$$\cos(h) = \cos(e - i) = \cos(e) \times \cos(i) + \sin(e) \times \sin(i)$$

$$\cos(p) = \cos(e + i) = \cos(e) \times \cos(i) - \sin(e) \times \sin(i)$$

If we add the two together:

$$\cos(h) + \cos(p) = \cos(e) \times \cos(i) + \sin(e) \times \sin(i) + \cos(e) \times \cos(i) - \sin(e) \times \sin(i)$$

Therefore:

$$\cos(h) + \cos(p) = 2 \cos(e) \times \cos(i)$$

And so...

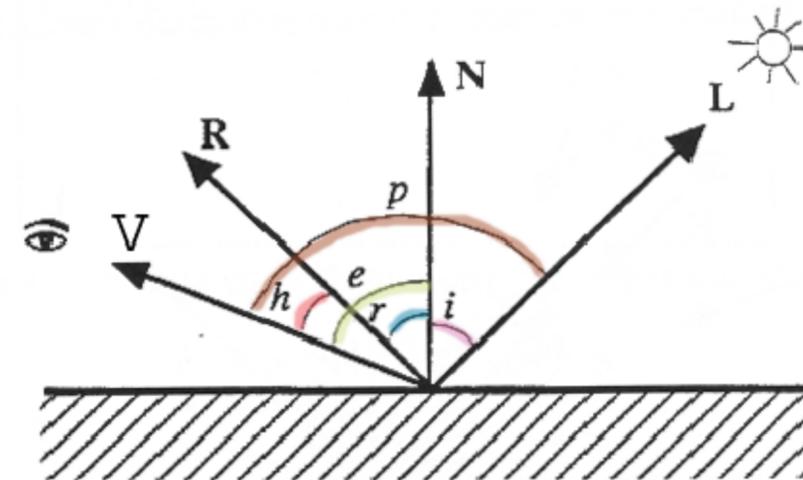
$$\cos(h) = 2 \cos(e) \times \cos(i) - \cos(p)$$

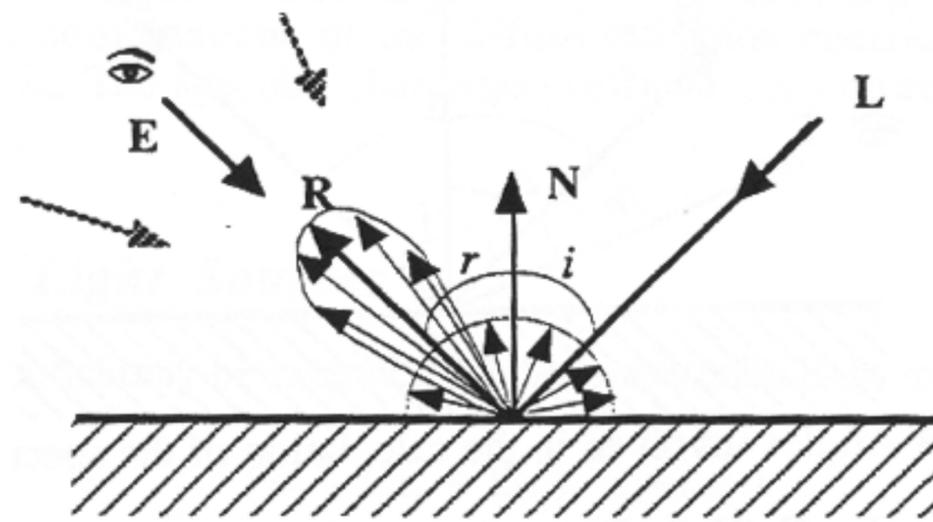
If we substitute known vectors into the equation:

$$\cos(h) = 2 \times (N \cdot V) \times (N \cdot L) - (V \cdot L)$$

Thus from our Phong equation:

$$R \cdot V = 2 \times (N \cdot V) \times (N \cdot L) - (V \cdot L)$$





$I_a$ =Ambient Intensity

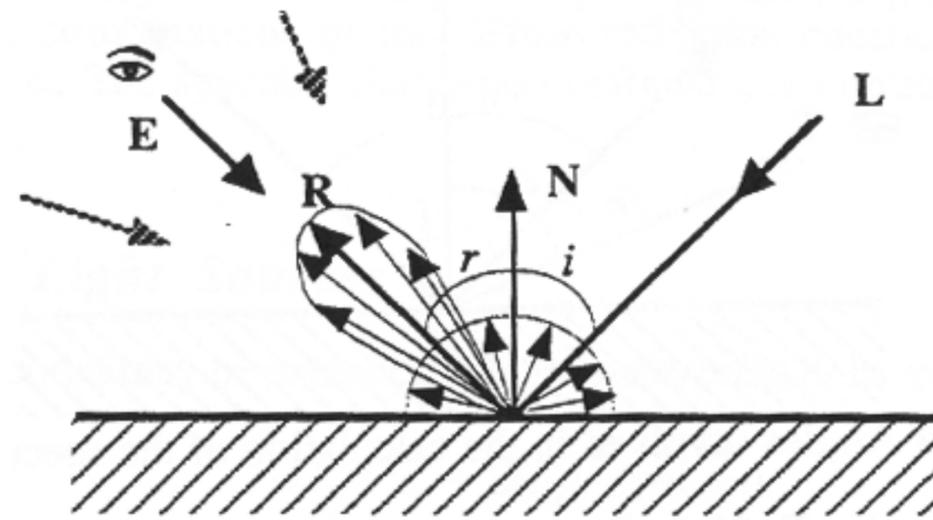
$I_p$ =Light Intensity

$K_d$ =diffuse coefficient

$K_s$ =specular coefficient

$$I = I_a k_a + f_{att} I_p k_d (N \cdot L) + f_{att} I_p k_s (R \cdot V)^n$$

Consider a light with intensity of 1 unit...

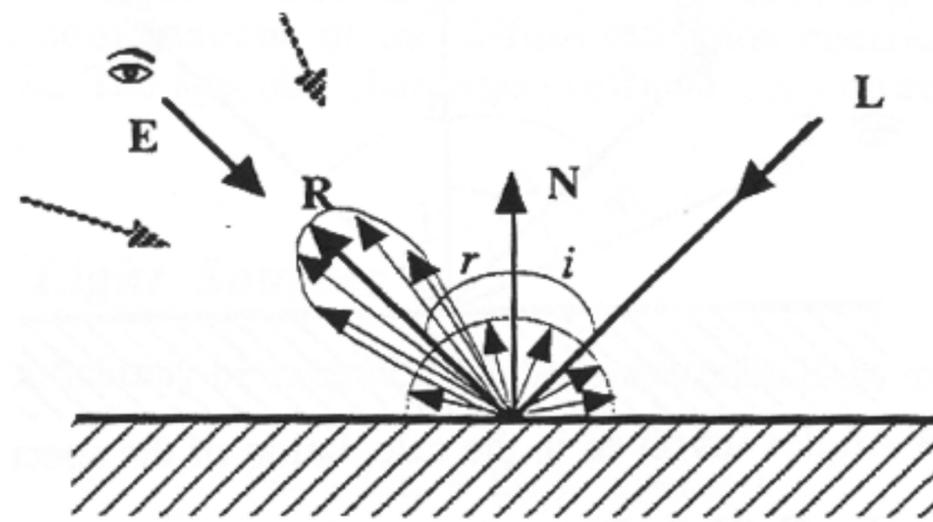


$I_a$ =Ambient Intensity  
 $I_p$ =Light Intensity  
 $K_d$ =diffuse coefficient  
 $K_s$ =specular coefficient

$$I = I_a k_a + f_{att} k_d (N \cdot L) + f_{att} k_s (R \cdot V)^n$$

Consider a light with intensity of **1 unit...**

Ignore light attenuation and ambient values completely...(I<sub>a</sub>=0, f<sub>att</sub> =1)



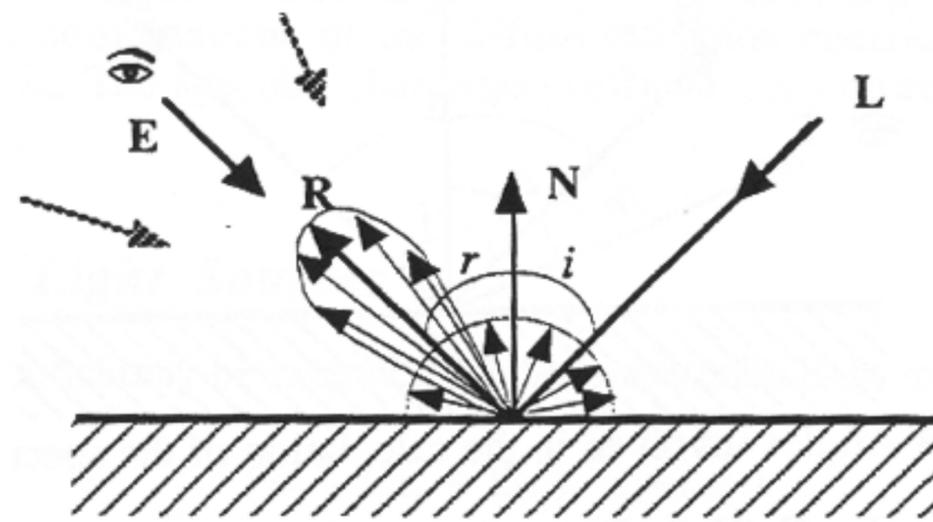
$I_a$ =Ambient Intensity  
 $I_p$ =Light Intensity  
 $K_d$ =diffuse coefficient  
 $K_s$ =specular coefficient

$$I = k_d (N \cdot L) + k_s (R \cdot V)^n$$

Consider a light with intensity of **1 unit...**

Ignore light attenuation and ambient values completely...(I<sub>a</sub>=0, f<sub>att</sub> =1)

$K_d$  and  $K_s=1 \dots$



$I_a$ =Ambient Intensity  
 $I_p$ =Light Intensity  
 $K_d$ =diffuse coefficient  
 $K_s$ =specular coefficient

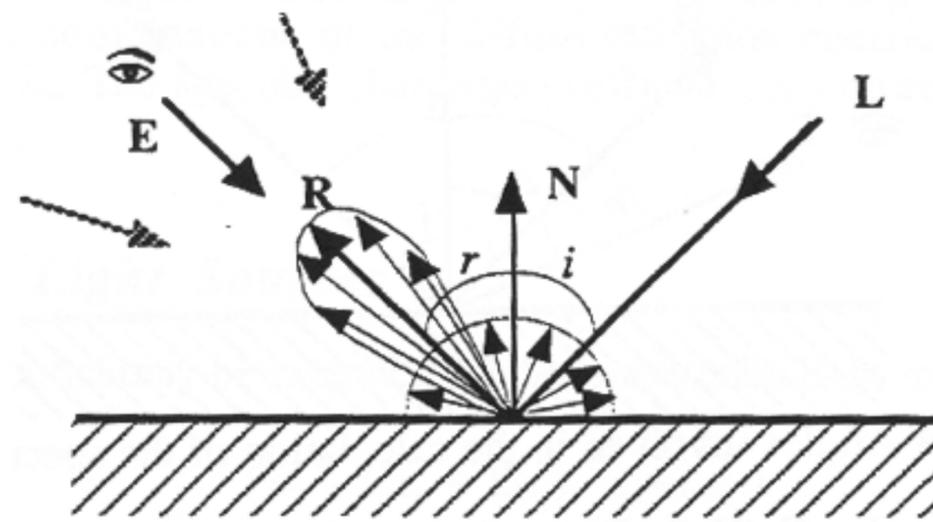
$$I = (N \cdot L) + (R \cdot V)^n$$

Consider a light with intensity of **1 unit...**

Ignore light attenuation and ambient values completely...( $I_a=0$ ,  $f_{att}=1$ )

$K_d$  and  $K_s=1$ ...

E directly opposed to R – we are looking at the optimum angle of incidence...



$I_a$ =Ambient Intensity  
 $I_p$ =Light Intensity  
 $K_d$ =diffuse coefficient  
 $K_s$ =specular coefficient

$$I = (N \cdot L) + 1$$

Consider a light with intensity of **1 unit...**

Ignore light attenuation and ambient values completely...( $I_a=0$ ,  $f_{att}=1$ )

$K_d$  and  $K_s=1$ ...

E directly opposed to R – we are looking towards the optimum angle of incidence...

What is the illumination when :

$$(N \cdot L) = 0.8$$

$$I = I_a k_a + f_{att} I_p k_d (N \cdot L) + f_{att} I_p k_s (R \cdot V)^n$$

$$I = 0 + 0.8 + 1$$

$$I = 1.8$$

More light is leaving the surface than is entering it! ?

Another adaptation with this model is to once again limit the intensity of that of the light.

$$(I_a k_a + f_{att} I_p k_d (N \cdot L) + f_{att} I_p k_s (R \cdot V)^n) \leq 1$$

# The Blinn Model

We are dealing with a surface with multiple facets.

This micro facet distribution held the key to a lot of further developments.

Blinn stipulated that if a surface was a collection of randomly distributed facets, the orientation of the best facet for specular reflection would be one where:

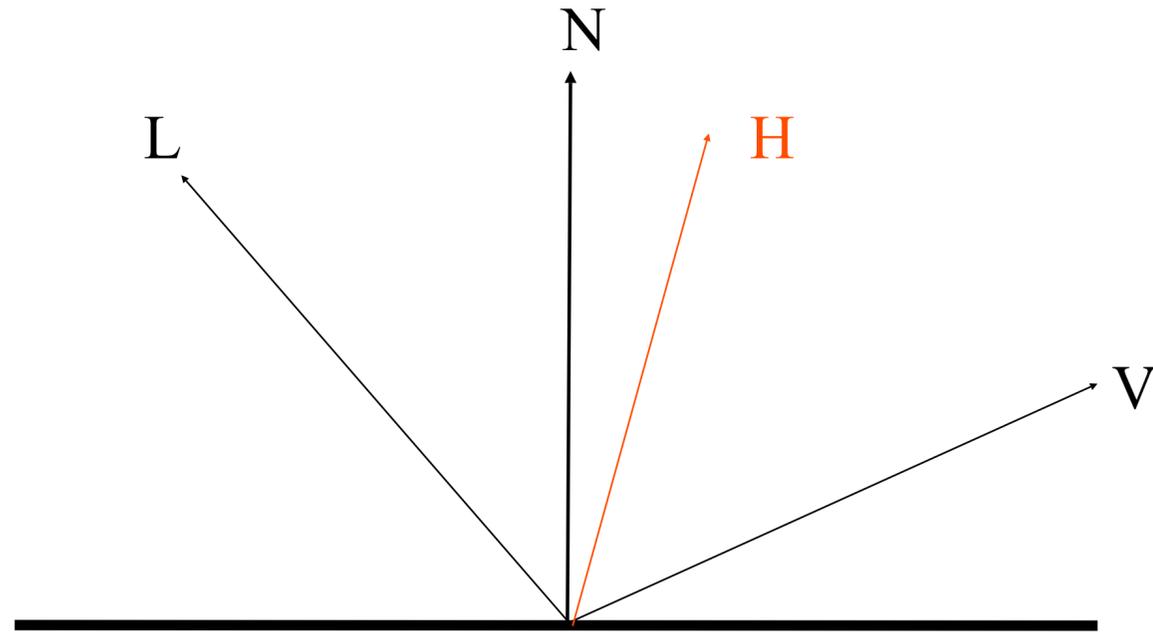
$$L \cdot N_B = V \cdot N_B$$

Where  $L$ =the light direction

$N_B$ =Blinn normal

$V$ =Eye/Viewer direction

# Blinn II



Thus Blinn proposed a new vector replace the normal to represent basis for specular reflection.

$$H = \frac{(L + V)}{2}$$

As the viewer approaches a view perpendicular to the surface normal of illumination, the specular highlights a proportionately larger than Phongs.

# Anisotropic Models

*unequal scattering of light*

Anisotropic functions take *direction* of micro facets into account.

These equations accommodate surfaces such as brushed metal where groves aligned in a given direction.

The equation usually takes advantage of UV coordinates (as surface derivatives, the rate of change in the surface as the current position).

These models take into account self shadowing in the groves to limit illumination, thus allowing a given direction and the viewer to dictate illumination.

# Cooke–Torrance 1981

Cooke and Torrance proposed a method whereby scattering of light is wavelength independent i.e. different coloured light behaves in different manners.

There are 3 functions that contribute to this model:

**Micro Facet Distribution**

**Geometric Attenuation**

**Fresnel**

# Micro Facet Distribution

E.g. based on a Beckman distribution function:

$$D = \frac{e^{-\left(\frac{\tan \beta}{m}\right)^2}}{4m^2 \cos^4 \beta}$$

Where

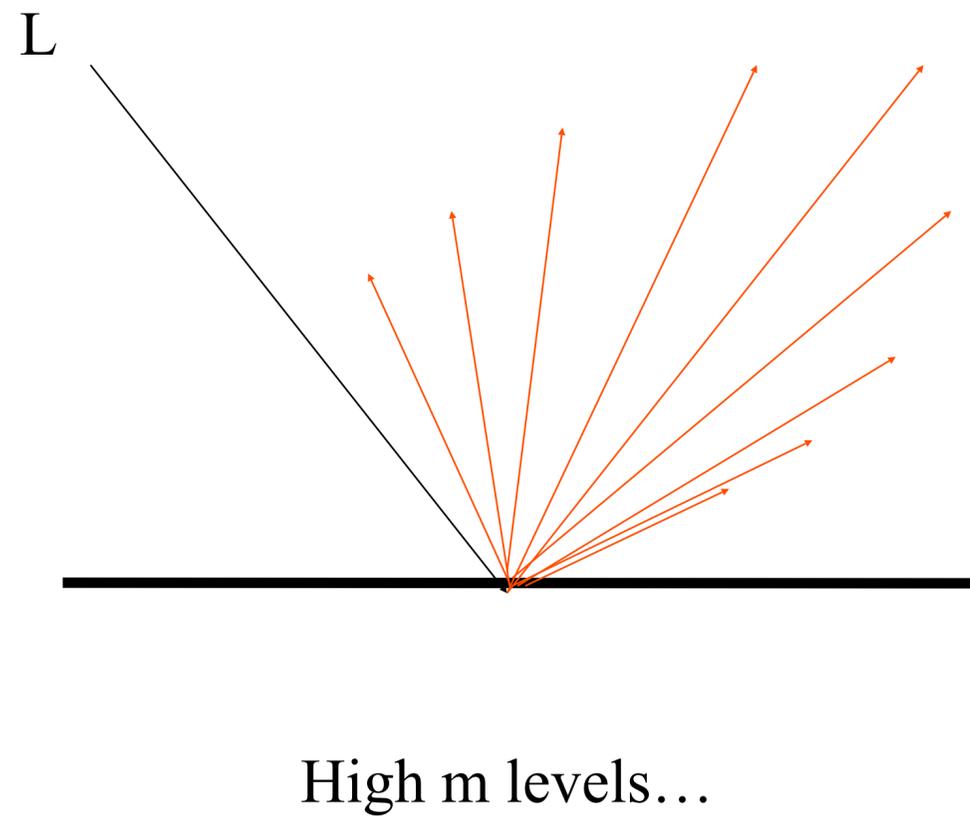
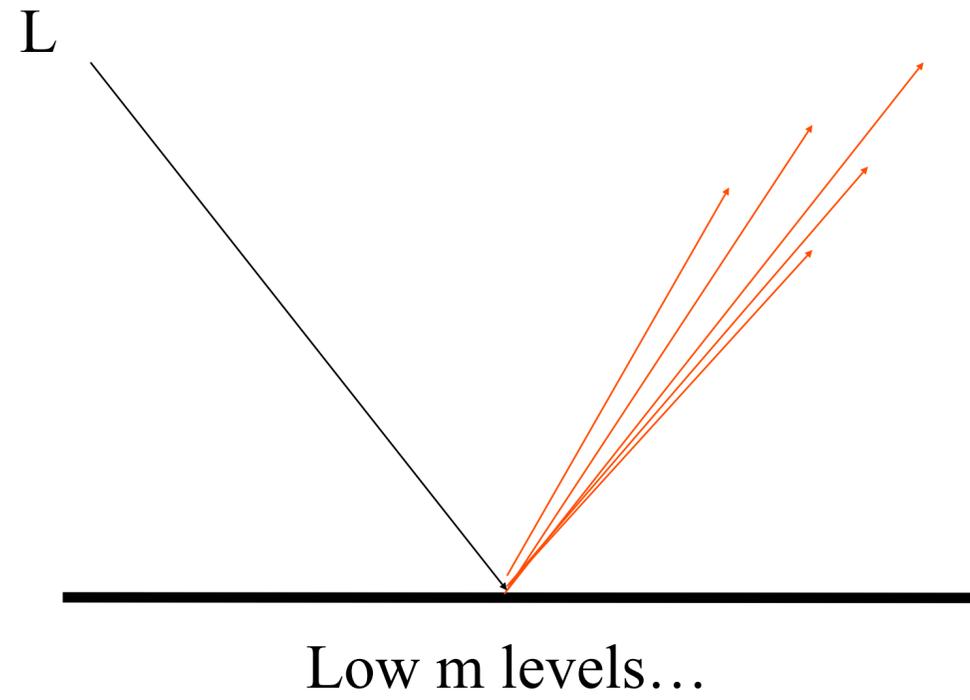
$\beta$  = the angle between N and H

m = the root-mean-square slope of the micro-facets.

Large m indicates steep slopes between facets(light spread out)

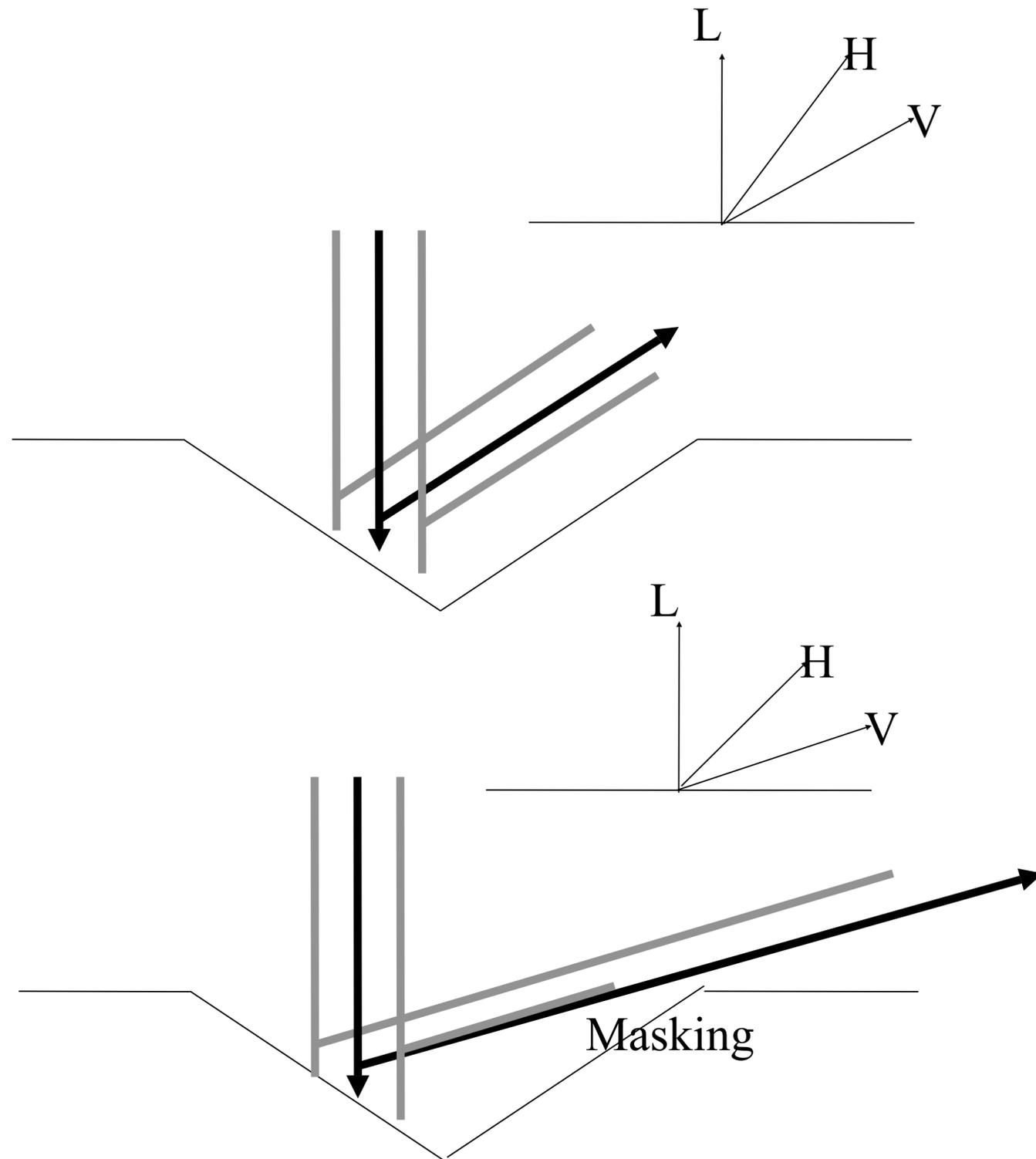
Small m indicates smaller falloffs

# Distribution



# Geometric Attenuation

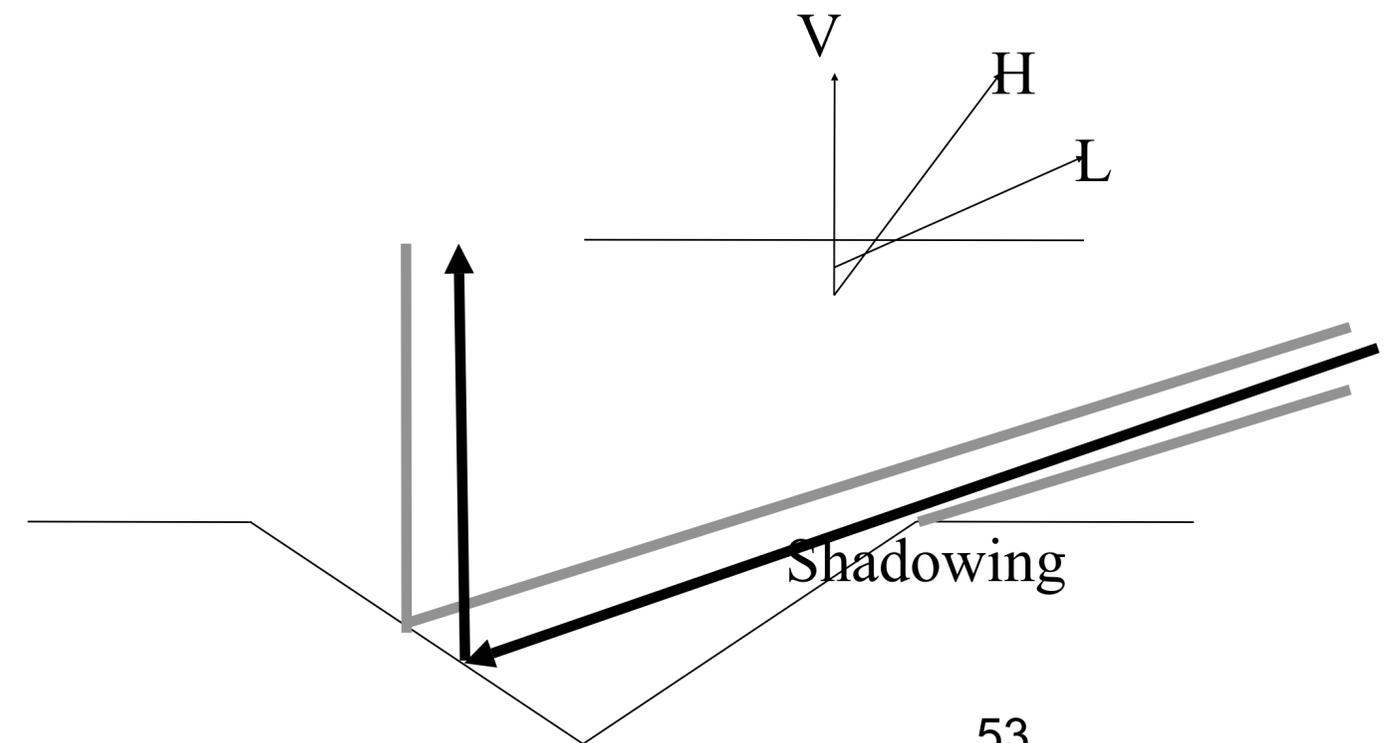
Accounts for shadowing and masking of micro facets by each other...



$$G_{masking} = \frac{2(N \cdot H)(N \cdot V)}{V \cdot H}$$

$$G_{shadowing} = \frac{2(N \cdot H)(N \cdot L)}{V \cdot H}$$

$$G = \min(1, G_{masking}, G_{shadowing})$$



# Fresnel

Light is an electromagnetic field.

The angle at which incoming light strikes a surface causes light to reflect in different manners.

This is due to the orientation of the electromagnetic field when it hits a surface.

A non-homogeneous material may have different reflectance for r,g and b

# Fresnel

$$F = \frac{1}{2}(\rho_1^2 + \rho_2^2)$$

$$\rho_1 = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

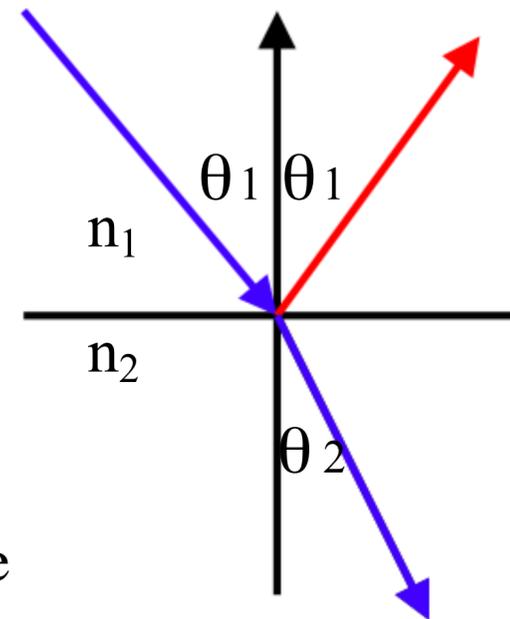
$$\rho_2 = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$\rho_1$  = reflect coeff. light parallel to plane of incidence

$\rho_2$  = reflect coeff. light orthogonal to plane of incidence

$n_1$ =index of refraction of environment

$n_2$ =index of refraction of medium



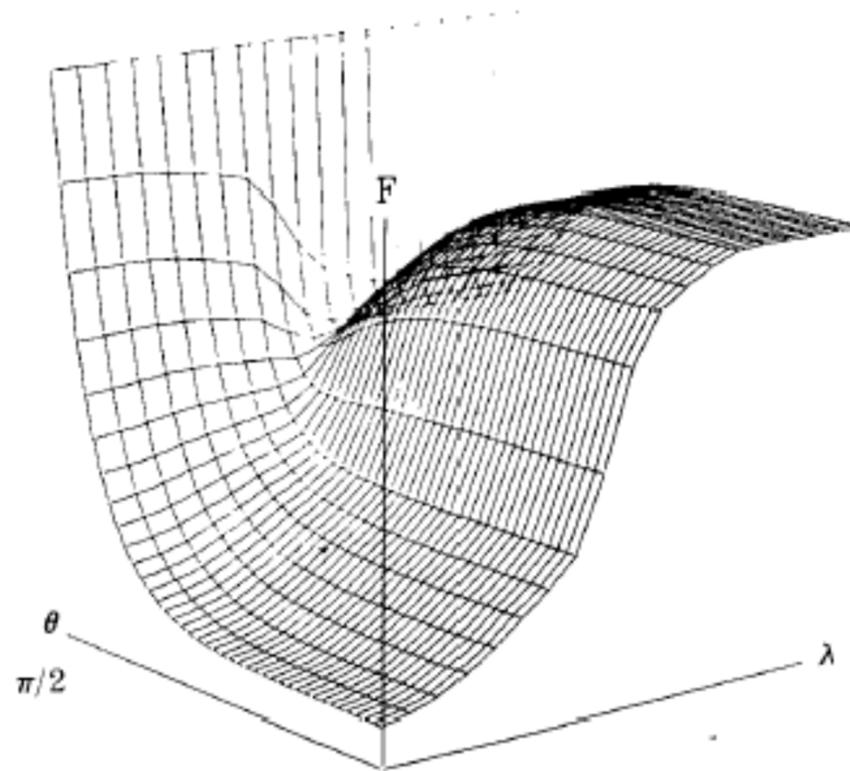
# Cooke Torrance Specular Reflection Equation

$$R = \frac{F}{\pi} \frac{D}{N \cdot L} \frac{G}{N \cdot V}$$

$$G = \min\left(1, \frac{2(N \cdot H)(N \cdot V)}{V \cdot H}, \frac{2(N \cdot H)(N \cdot L)}{V \cdot H}\right)$$

$$D = \frac{e^{-\left(\frac{\tan \beta}{m}\right)^2}}{4m^2 \cos^4 \beta}$$

# Reflectance Function



The reflectance of copper as a function of wavelength and incidence angle.

# BSSRDFs

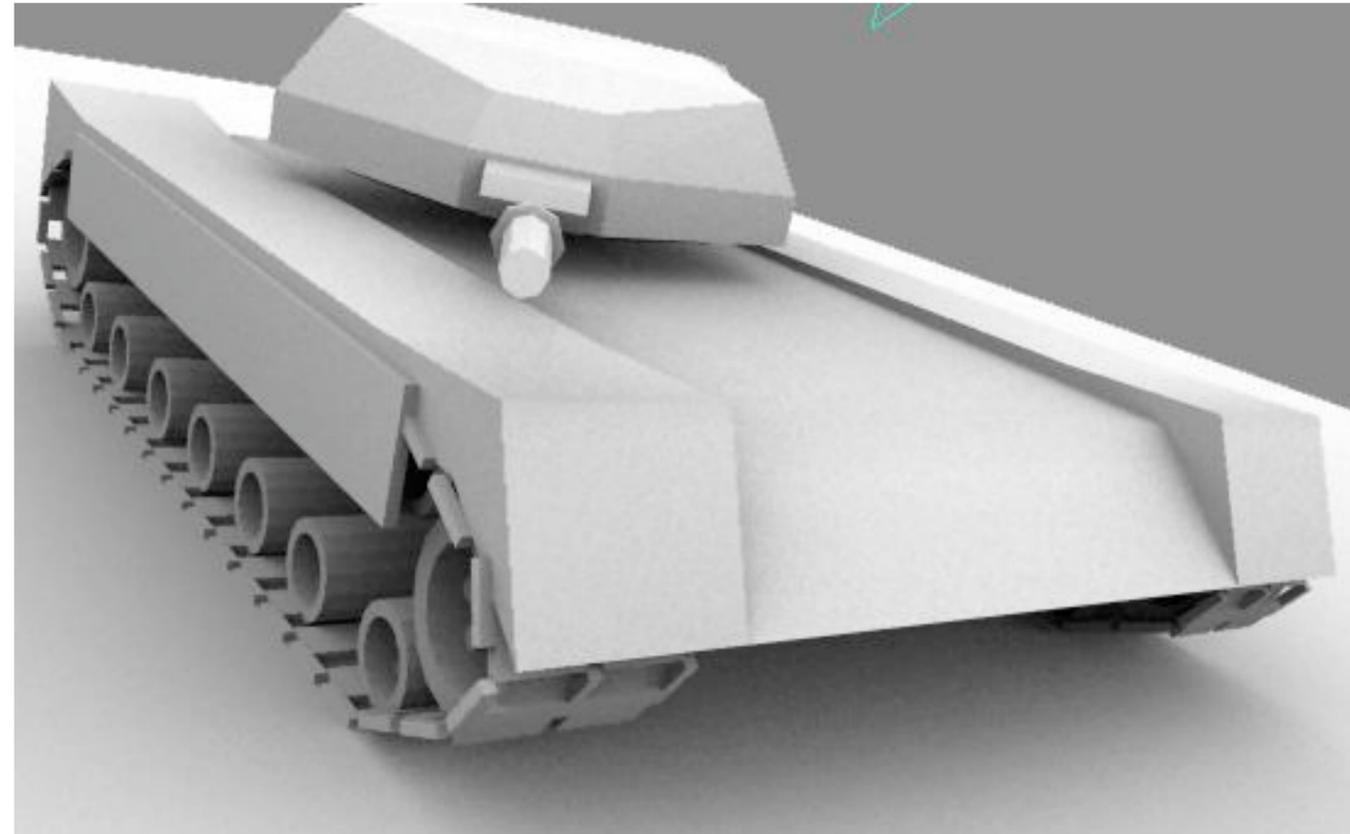
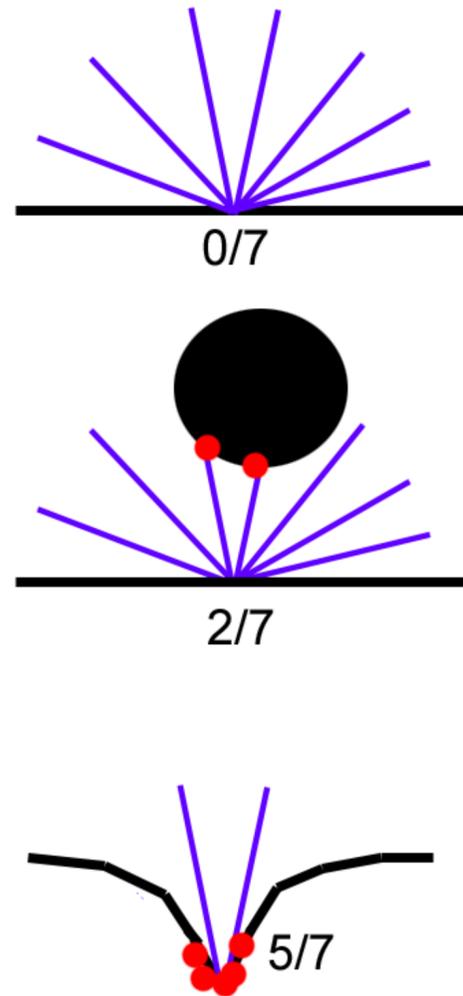
*bidirectional scattering surface reflection distribution function.*

which account for light that scatters on and through a given surface.

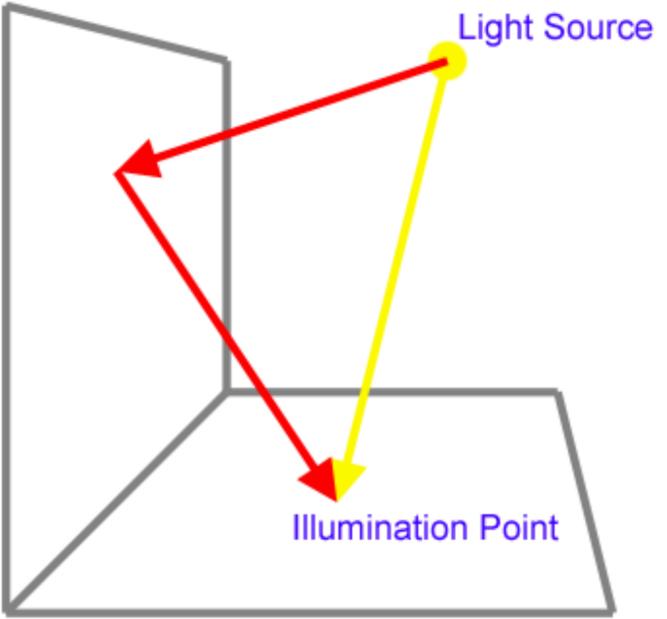
They take into account incoming position and direction and outgoing position and direction. (bidirectional sub surface scattering distribution functions).

# Ambient Occlusion

The **probing** of space around a render point to determine the ratio of surface occlusion



# Global Illumination



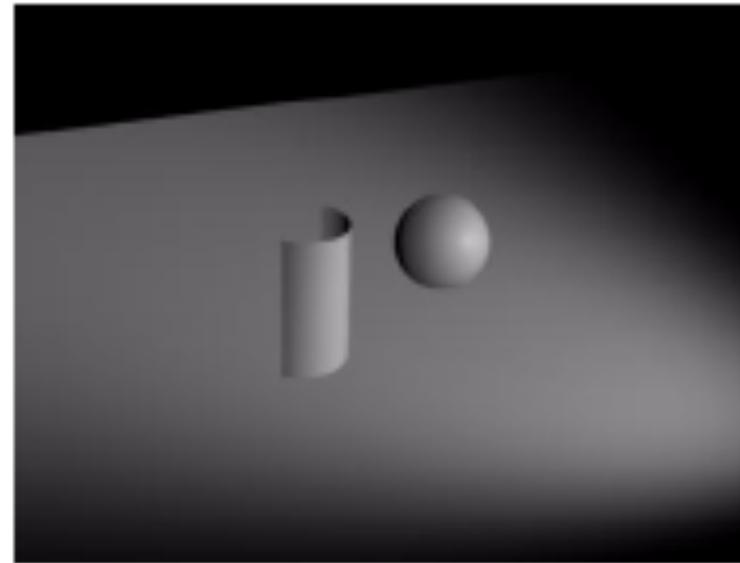
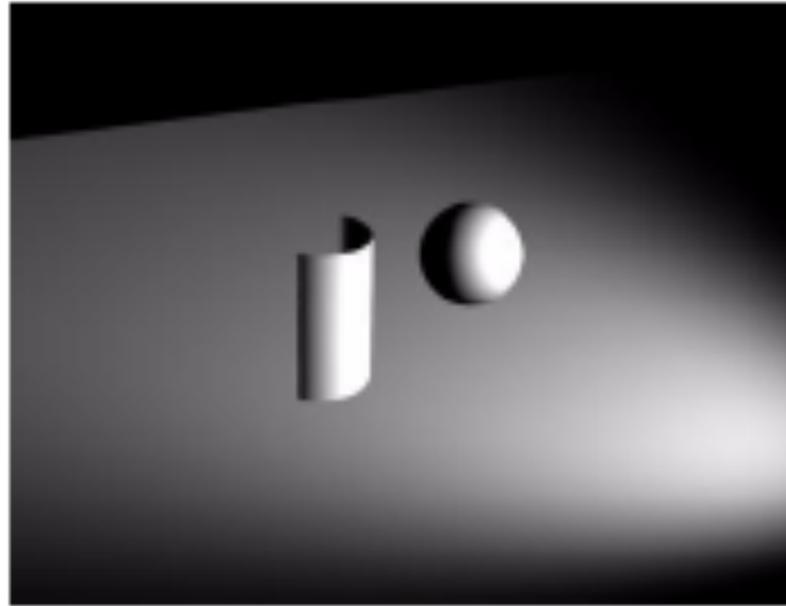
# Modifications



You can change anything.

Sony Image Pictureworks played with illumination in “Stuart Little” so that light intensity decayed over 120 degrees, and not the usual 90 degrees.

This technique gave a softer light to the character – effectively light was travelling around corners.



For more info, see “To RI\_INFINITY and beyond” – Siggraph 2000

# References

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- <http://en.wikipedia.org/wiki/Brdf>
- [http://graphics.cs.ucdavis.edu/~bcbudge/ecs298\\_2004/](http://graphics.cs.ucdavis.edu/~bcbudge/ecs298_2004/) (Randall Rauwendaal BRDF.ppt)
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