National Centre for Computer Animation

2D Mathematical Foundations 1

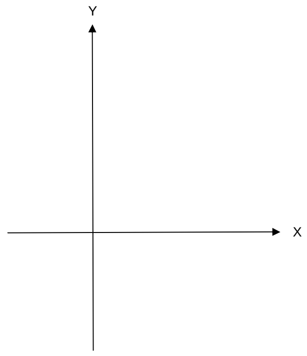
Do not disturb my circles!
(*Archimedes*)

ES Overview

- Introduction to ES
- **2D Graphics in Entertainment Systems**
- Sound, Speech & Music
- 3D Graphics in Entertainment Systems

Cartesian Coordinates

- named after René Descartes
- one of the most commonly used coordinate systems
- 2D rectangular coordinate system, i.e. defined by two axes that are at right angles (*orthogonal/perpendicular*) to each other
- axes are labelled x and y, creating the xy-plane



Points

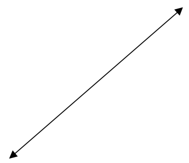
A point is a location in space with an infinitely small extent.

- Points denominate positions in space in reference (*anchored*) to the origin of the coordinate system used.
- Their position within the coordinate system is indicated as an ordered pair (***x,y***), i.e. the abscissa (*x-coordinate*) followed by the ordinate (*y-coordinate*).

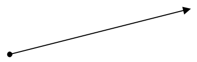
Lines

a

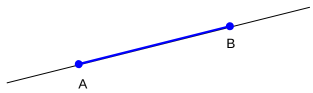
- line
  - is an infinitely thin, infinitely long (*straight*) geometrical object
    (*given 2 points, there is always exactly one line that passes trough them, providing the shortest connection between them*)



- ray
  - is a constrained (*half of a*) line which starts at one point and extends infinitely far in only one direction

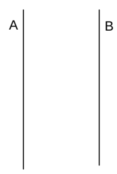


- line segment
  - is a constrained line which starts at one point and ends at another point
    (*usually named after its end points*)

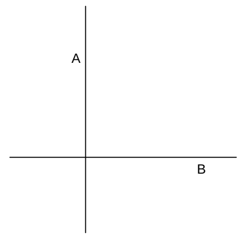


Lines

In 2D space, two lines that do not intersect are said to be **parallel**
(*notation: A*‖*B*)



Two lines are said to be **perpendicular** if they meet at a right angle
(*notation: A*⊥*B*)



Lines

Equation of line:

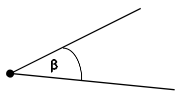
$$y = mx + c$$

- ***m*** is the slope of the line
- ***c*** is a constant which is the y-intercept of the line

Angles

An angle is formed between two rays that share a common starting point.

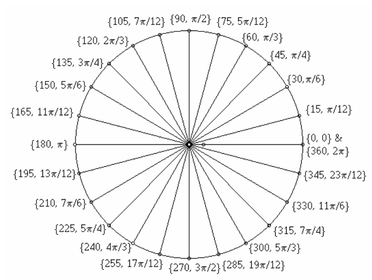
- A degree is a measurement unit for angles derived from defining a complete rotation as 360°
- As the perimeter of a circle is $2\pi r$, a complete rotation equals 2π rad (*radians*)



$\pi/2=90°$ $\pi=180°$ $3\pi/2=270°$ $2\pi=360°$

radians = degrees x $\pi/180$

degrees = radians x $180/\pi$



Degree to Radians Conversions

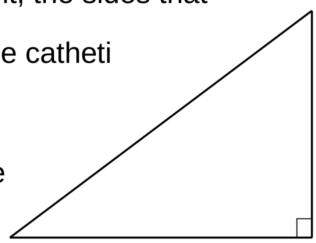
From: http://math.rice.edu/~pcmi/sphere/drg_txt.html

Triangles

All 3 angles in a triangle add up to 180°

In a triangle with a right angle in it, the sides that form the right angle are called the catheti of the triangle.

The side opposite the right angle is called the hypotenuse.

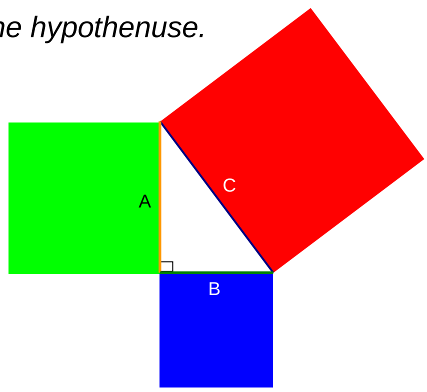


Pythagorean Theorem

Named after the Greek mathematician Pythagoras.

The sum of the square values of the two catheti is equal to the square value of the hypothenuse.

$$C^2 = A^2 + B^2$$



$\rightarrow$ $C = \sqrt{(A^2+B^2)}$

Pythagoras Example

Using Pythagoras we can calculate the distance between 2 points:

Points: A (2,1) B (5,3)

length of blue line (x-direction): $5-2 = 3$

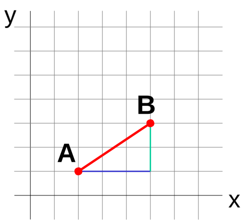
length of turquoise line (y-direction): $3-1 = 2$

distance between A and B: $\sqrt{(3^2+2^2)}$

$= \sqrt{(9+4)}$

$= \sqrt{13}$

generalised: $d = \sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$

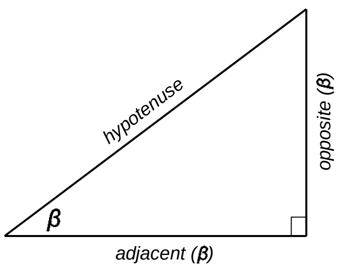


Trigonometric Ratios

Sine (*sin*) - ***sin(β)* = *opposite/hypotenuse***

Cosine (*cos*) - ***cos(β)* = *adjacent/hypotenuse***

Tangent (*tan*) - ***tan(β)* = *opposite/adjacent***



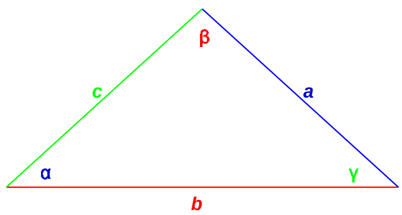
Sine Rule

- useful to discover the length of unknown sides of a triangle if only two angles and one side are known
- commonly used for triangulation

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

$\leftrightarrow$

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$



Sine Rule Example

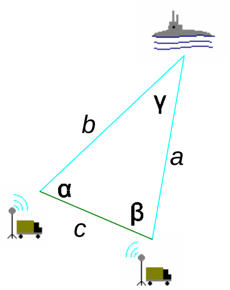
triangulation: determine an unknown distance by measuring two angles and a distance

given: $\alpha = 50°$, $\beta = 100°$, $c = 10$

we know that $\gamma = 180° - (\alpha + \beta) = 30°$

sine rule: a / sin(50) = 10 / sin(30)

therefore: a = 10 sin(50) / sin(30) = 15.321



Cosine Rule

Useful to discover the missing data of a triangle if only two sides and one angle are known (*only results in a unique triangle if that angle is contained between the two known sides*)

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$b^2 = c^2 + a^2 - 2ca\cos(\beta)$

$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

$a = b\cos(\gamma) + c\cos(\beta)$

$b = c\cos(\alpha) + a\cos(\gamma)$

$c = a\cos(\beta) + b\cos(\alpha)$